IIT DHARWAD AUTUMN 19

Engg. Maths for Advanced Studies

MODULE - VECTOR CALCULUS

QUIZ 01 - SOLUTIONS

Q.1

(a) $u \times v = v \times u$ FALSE

(b) $u \cdot (v \times w) = v \cdot (u \times w)$ FALSE

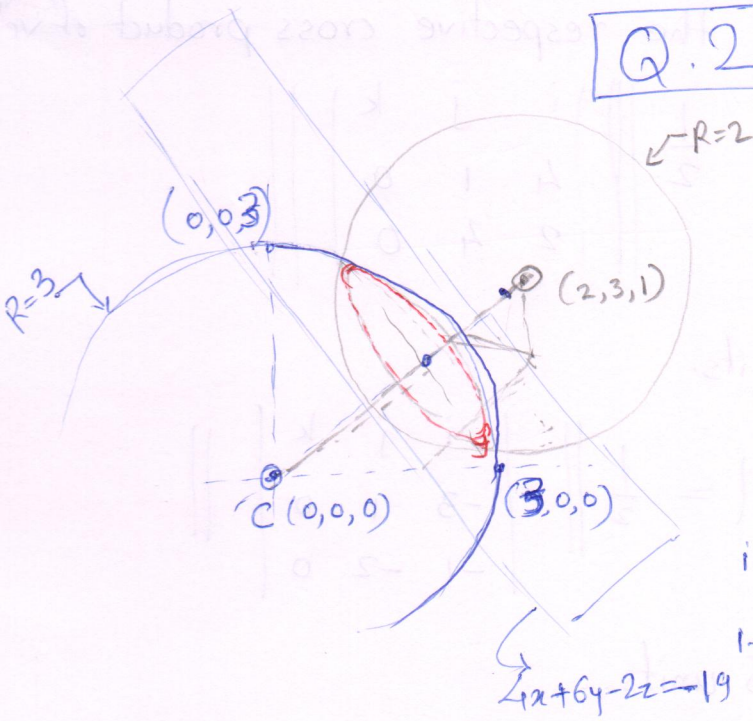
(c) $w \cdot (v \times u) = -w \cdot (u \times v)$ TRUE — (Note that the typo was clarified during exam).

(d) $w \cdot (v \times u) = v \cdot (u \times w)$ TRUE

(e) $\|v + w\| = \|v\| + \|w\|$ FALSE

(f) $u \cdot v = v \cdot u$ TRUE.

Q.2



Sphere 1 →

$x^2 + y^2 + z^2 = 9$

Sphere 2 →

$(x-2)^2 + (y-3)^2 + (z-1)^2 = 4$

i.e. $x^2 - 4x + 4 + y^2 - 6y + 9 + z^2 - 2z + 1 = 4$

i.e. $x^2 + y^2 + z^2 - 4x - 6y - 2z + 10 = 0$

$= 9$ for common shared points.

$\therefore$ **$4x + 6y - 2z = -19$**

↳ plane containing red circle. i.e. common points.

Geometry of shared locii will be a circle as it is intersection of two spheres

~~As~~ Computation of position vectors of the shared locii is somewhat involved & hence not considered for grading

Q.3

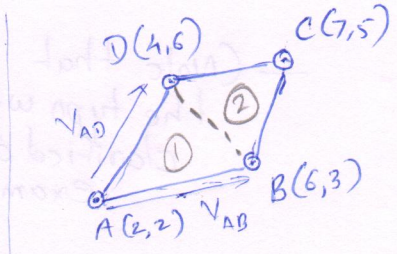
$\vec{p} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

$\vec{v} = 4\hat{i} - \hat{j} + 6\hat{k}$

∴ eqn. of Line through P along $\vec{v}$

$\vec{p} + \alpha\vec{v}$ where $-\infty < \alpha < \infty$

Q.4



$V_{AB} = 4\hat{i} + \hat{j}$

$V_{AD} = 2\hat{i} + 4\hat{j}$

$V_{CD} = -3\hat{i} + \hat{j}$

$V_{CB} = -\hat{i} - 2\hat{j}$

Area of ① & ② are $\frac{1}{2}$ of the respective cross product of respective vectors

Area ① $= \frac{1}{2}\|(\vec{V}_{AB} \times \vec{V}_{AD})\| = \frac{1}{2}\left\|\begin{vmatrix} i & j & k \\ 4 & 1 & 0 \\ 2 & 4 & 0 \end{vmatrix}\right\|$

$= \frac{1}{2}\|14\,\hat{k}\| = 7$ units.

Area ② $= \frac{1}{2}\|(\vec{V}_{CD} \times \vec{V}_{CB})\| = \frac{1}{2}\left\|\begin{vmatrix} i & j & k \\ -3 & 1 & 0 \\ -1 & -2 & 0 \end{vmatrix}\right\|$

$= \frac{1}{2}\|7\,\hat{k}\| = 3.5$ units.

∴ Total Area = ~~10~~ $7 + 3.5 = 10.5$ sq. units.

∴ Total Crop produce = $10.5 \times 10 = 105$ units

Q.5

$$r(t) = a\cos(t)\,\hat{i} + b\sin(t)\,\hat{j} + c(t)\,\hat{k}$$

(a) Elliptical Spiral

(Note - typo correction $c(t) = t$.)

(b) $r'(t) = -a\sin(t)\,\hat{i} + b\cos(t)\,\hat{j} + \hat{k}$

$\therefore$ At $t=2$

$$r'(t=2) = -a\sin(2)\,\hat{i} + b\cos(2)\,\hat{j} + \hat{k}$$

$$\therefore \hat{e}_{r'} = \frac{1}{\sqrt{1 + a^2\sin^2(2) + b^2\cos^2(2)}}\left(-a\sin(2)\,\hat{i} + b\cos(2)\,\hat{j} + \hat{k}\right)$$

(c) $$\int_1^2 r'\cdot r'\,dt = \int_1^2 \left(a^2\sin^2(t) + b^2\cos^2(t) + 1\right) dt$$

$$= \int_1^2 \left(\cos^2(t)\left(a^2+1\right) + \left(b^2-a^2\right)\cos^2(t)\right) dt$$

$$= \left(a^2+1\right) + \left(b^2-a^2\right)\int_0^2 \cos^2(t)\,dt$$

Q.6

(a) ~~FALSE~~ ~~TRUE~~ — to be specific "magnitude" of component of $\vec{a}$ along $\vec{p}$. only true if $\|\vec{p}\| = 1$

(b) FALSE

(c) TRUE