Vector calculus - Assignment #1

Engineering Mathematics for Advanced Studies
IIT Dharwad
Autumn 2019

Submission - Tuesday 15th Oct. 2019 5:30pm
Total Score -10 marks
Late penalty - 1 day late* 30%, 100% for more than a day (*starts from 5:31pm, 15th Oct. 2019!)

1. Give the equation of a line passing through point $P(2,3,4)$ and parallel to the direction $\vec{v}(2,1,2)$:

   (Hint: refer to similar problem 1.19 in Michael Corral Vector Calculus notes redirected on course portal)

   (a) Vector form?

   (b) Parametric form?

   (c) Symmetric form?

   (d) What is the angle between position vector of P say $\vec{r_p}$ and the position vector of point Q , say $\vec{r_q}$ along the vector direction $\vec{v}$ on the above line at distance 4 units from P ? (marks 2)

2. For the upcoming festival season, we need to decorate an elliptical pillar with string lights decoration which goes around the pillar spiralling. The major axis of elliptical cross section of pillar is 50 cm and the minor axis is 40 cm. Height till the roof is 500 cm. Spiralling starts at the minor axis of the beam at the bottom of pillar and continues spiralling with pitch length of 40 cm and ends at the top of the pillar. (Spiralling is assumed to have axial progress proportional to the angular movement about the axis of the pillar) (marks 8)

   (a) What is the parametric vector expression for position vector $\vec{r}(\theta)$ to locate any points on the string where is θ is angular displacement?

   (b) What is the range of the θ in above example?

   (c) What is the length of the string light needed?

   (d) What is unit tangent vector at the first major axis and minor axis helically wound the light string?

   (e) What is the curvature and torsion and slope values $\frac{dz}{d\theta}$ at a point near tip of the minor axis?

   (f) What is the curvature and torsion and slope values $\frac{dz}{d\theta}$ at a point near tip of the major axis?

   (g) Can you express $\vec{r}$ as $\vec{r}(s)$ where s is arc length along light string? (exploratory work, may need elaborate work)

   (h) Verify if tangent $\frac{d\vec{r}(s)}{ds}$ tallies with the answer in (d)? (exploratory work, may need elaborate work)