IIT DHARWAD AUTUMN 2019

Engg. Maths for Advanced Studies

Sample Solutions: Probability & Statistics
Final Exam

Q.1

Answer → (a) $P(E \cup G) = P(E) + P(G)$

(b) $P(E \cup G) = P(E) + P(G) \mp P(E \cap G)$

key underlying fact (KUF) → E & G are mutually exclusive & hence $P(E \cap G) = 0$.

Q.2

(a) (i) $(10^4-1)(10^2-1)$

KUF → "AB pq XXXX" has 3 parts AB, pq & XXXX.

AB → Given that fixed set of alphabets for 'AB' in a fixed sequence are predesignated for the state, possible variations are none – only one possibility

pq → 'pq' has two digits each of which can be one of the ten possibilities 0-9. Hence, total $10 \times 10 = 10^2$ possibilities out of which ignore the one in which both places are zeros. Hence (10^2-1)

XXXX → 'XXXX' will have (10^4-1) using same rationale as above.

Q.2

(b) $(45)(10^4-1)$.

Q.3

3 independent trials where each has possibility 0.5 of getting a head. (Also, note that there is only one permutation possible).

$$\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)\times(1) = \frac{1}{8}.$$

(e.g. if question was 1 head & 2 tails we should add up individual probabilities of all three permutations -: HTT, THT, TTH.

i.e. $\frac{1}{8}+\frac{1}{8}+\frac{1}{8} = \frac{3}{8}.$).

Q.4

(a) Poisson's distribution is approximation to binomial distribution as number of trials goes to infinity.

Q.5

Answer → (a) → (E & F are mutually exclusive) → NOT possible

KUF → $P(E\cup F) = P(E)+P(F) - P(E\cap F)$.

LHS has to be less than ~~zero~~ equal to one as its a probability

& Hence

$$P(E)+P(F)-P(E\cap F) \leq 1$$

$\therefore$ $0.3 + 0.8 - P(E\cap F) \leq 1$

$\therefore$ $0.1 \leq P(E\cap F)$

i.e. E & F can not be mutually exclusive.

Q.5

Other comments. @ why other options can be true!

(b). since P(F) & P(G) sum up to one, it is possible that $P(F \cap G) = 0$ i.e. F & G are mutually exclusive

(c) & (d) Independent events imply only that joint possibility ~~is~~ i.e. say $P(E \cap F)$ is multiplication of $P(E)\,P(F)$
There is not enough information to rule out independence

Q.6

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$P(E \cap F)$ → probability that both event ~~can~~ happen

$P(F)$ → probability of event F
i.e. a number less than or equal to one is in denominator

Answer: (d) $\geq$

$\therefore P(E|F) \geq P(E \cap F)$.

Q.7

Answer → (a) $X = \{x : -10 \leq x \leq 10\}$

Q.8

Answer → (c) $Var(X) = E((X-\mu)^2) = E(X^2) - \mu^2$

Q.9

Answer → <u>False</u> ~~GG~~ CG → $E(X) = \int x \cdot p(x)$ → mean.

M.I → Variace → $E((X-\mu)^2) = \int (x-\mu)^2 \cdot p(x)$.

moment of inertia ← M.I @ mean value

second moment @ μ

Session B - Open Book

$\boxed{Q.1}$

	event A^c	event A	
event B	4500	2000	6500
event B^c	3000	500	3500
	7500	2500	

(a) $\dfrac{2000}{6500} = \dfrac{4}{13}$

(b) $\dfrac{2000}{2500} = \dfrac{4}{5}$

(c) $\dfrac{2500}{7500+2500} = \dfrac{2500}{10000} = \dfrac{1}{4}$

(d) $\dfrac{6500}{6500+3500} = \dfrac{6500}{10000} = \dfrac{13}{20}$

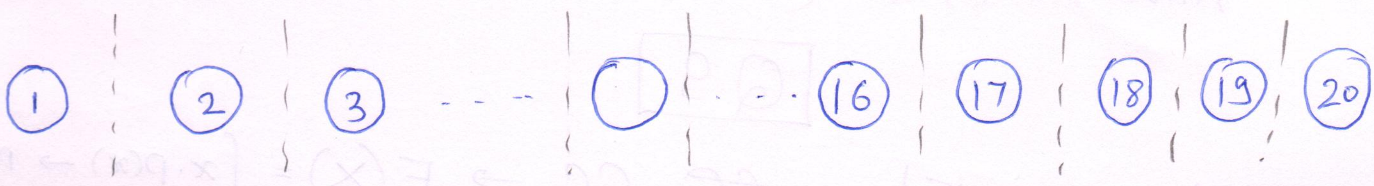
$\boxed{Q.2}$

This problem is similar to splitting 20 balls in 5 baskets.

(a) If each start-up is to get at least one crore.

Consider each unit of a crore as a labeled unit.

We need to put ~~5~~ $5-1=4$ divisors in 19 gaps.



- - - - Possible 19 locations to split ~~in 5~~ 1 - 20 in 5 segments.

Redefined problem: Select ~~5~~ 4 of 19 → ~~$^{19}C_5$~~ $^{19}C_4$ possible ways (non-empty)

(b) When empty set is allowed, i.e. one or more start-ups can be rejected any funding

If non-empty ~~solution to~~ integer solution of equation is given by

$$x_1 + x_2 + \cdots + x_r = n$$ where r = number of baskets. i.e. number of start-ups here

then empty+ solution can be given by replacing $x_i + 1 = y_i$ & hence $n + r$ will be RHS. One can imagin that there are additional r empty units of money (instead of 1 crore.

$$y_1 + y_2 \cdots + y_r = n + r.$$

$\therefore$ There are $\binom{n+r-1}{r-1}$ i.e. $\binom{20+5-1}{5-1}$ i.e. $24C_4$

$\therefore$ $\boxed{\text{Answer} \Rightarrow 24C_4}$

(c) If each start-up is to ~~remove~~ receive 2 crores each, it is redefined problem of distribution where 1 crore is already alloted to 5 start up companies. i.e. $20 - 5 = 15$ crores to be ~~re~~ distributed where non-empty distribution condition is enforced.

i.e $n' = 15$, $r = 5$

$\therefore$ Answer $\rightarrow$ $\binom{15-1}{5-1} = 14C_4$

Q.3

$D \rightarrow$ person has disease

$E \rightarrow$ lab test comes out to be positive.

We need $\Rightarrow P(D/E)$.

$$P(D/E) = \frac{P(DE)}{P(E)} = \frac{P(D) \cdot P(E/D)}{P(E)} \quad -(3.1)$$

$\because \quad P(E/D) = \frac{P(DE)}{P(D)}$.

$\therefore \quad P(DE) = P(D) \cdot P(E/D)$.

Also, Bayes formular to expand the denominator $P(E)$

$$P(E) = P(E/D) \cdot P(D) + P(E/D^c) \cdot P(D^c).$$

$$= (0.95)(0.005) + (0.01)(0.995)$$

$$= 0.0147$$

$P(D) = 0.005$

$P(E/D) = 0.95$

$\therefore$ substituting in above eqn. (3.1)

$$P(D/E) = \frac{(0.005)(0.95)}{0.0147}$$

$$\boxed{P(D/E) = 0.3231}$$

Q. 4

(a) Total number of students $= 5 + 6 = 11$.

Hence, possible permutations $\boxed{= 11!}$

(b) 5 MS students can be ranked in $5!$ way.

& 6 PhD students can be ranked in $6!$ way

$\boxed{\text{Total number of ranking possibilities} = 5! \cdot 6!}$

Q. 5

(a) Lets say student A & student B are the PhD students who can not be on the commitee simultaneously.

For selecting 2 students from 5 MS students we have $5C_2$ possibilities.

For selecting 2 students from 6 PhD students we have $6C_2$ possibilities. However we need to eliminate one possibility when both A & B are ~~st~~ selected. i.e. $(6C_2 - 1)$.

$\therefore$ Total number of possibilities $= (5C_2)(6C_2 - 1)$

$$= \left(\frac{5!}{3!\,2!}\right)\left(\frac{6!}{4!\,2!} - 1\right)$$

$$= (10)(15 - 1)$$

$$= 140$$

$\boxed{\text{Answer} : (5C_2)(6C_2 - 1) = 140}$

Q.6

Concise form of problem needs to decided first.

(a) $p = 0.7$, $n = 15$ for first win by daughter.
$\leftrightarrow (30/2)$.

Assuming ~~are~~ all results are independent, first 14 times we would need father to win. & last one be the win of daughter (which has $1-p = 1-0.7 = 0.3$ probability).

$\therefore$ Required probability $= (0.7)^{14}(0.3)$

(b) $p = 0.7$, $n = \frac{30}{2} = 15$ & we need $k = 10$ times win by daughter.

Out of total 15 times there are $15C_{10}$ i.e. $\binom{15}{10} = \frac{15!}{5!\,10!}$ ways in which daughter can win 10 games.

For any specific combination there are 5 wins by father & 10 losses by $(p_{loss} = 1 - P_{win} = 0.3)$ by father.

$\therefore$ Probability of that specific win-loose sequence $= (0.7)^5(0.3)^{10}$

Considering all possibilities of win-loose sequence variations for 10 wins by daughter

Total probability $= (15C_{10})(0.7)^5(0.3)^{10}$

$$\boxed{\text{Answer} = (15C_{10})(0.7)^5(0.3)^{10}}$$

$\boxed{Q.7}$

Let E_{10} be event that customer selects 10 ft room.
E_{20} - - - - - " ———— 20ft room
E_{30} —— - " ———— 30ft room
E_{40} —— - " ———— 40 ft room.

for room 10ft, area $= 10^2 = 100$ sq. ft.

$\therefore$ cost $= \cancel{\$}\ R_{10} = 100 \times 500 = 50000$ Rs.

Similarly $R_{20} = \cancel{\$}\ \cancel{40}\ (20)^2 \times 500 = 200000$ Rs.

$R_{30} = (30)^2 \times 500 = 450000$ Rs.

$R_{40} = (40)^2 \times 500 = 800000$ Rs.

with $E_{10} = 0.2$, $E_{20} = 0.4$, $E_{30} = 0.3$, $E_{40} = 0.1$

the expected value for the budget towards freebies.

$$E(R) = \sum_{i=1}^{4} P(E_i) \cdot R(E_i)$$

$$= (0.2)(\overset{50000}{\cancel{200000}}) + (0.4)(200000)$$

$$+ (0.3)(450000) + (0.1)(800000)$$

$$\boxed{E(R) = 3{,}05{,}000 \text{ Rs.}}$$