IIT DHARWAD AUTUMN 19

Engg. Mathematics for Advanced Studies

MODULE – Probability & Statistics.

Assignment 02 → Sample Solutions

Q.1

MATHEMATICS ⟶ M A T H E I C S

(M: 2, A: 2, T: 2)

Total ~~distinct~~ count = 11.

∴ Possible "Distinct" Permutations $= \frac{11!}{2!\,2!\,2!}$

- 11! → on account of all alphabets
- first 2! → on account of repeated 'M'
- second 2! → on account of repeated 'A'
- third 2! → on account of repeated 'T'

Q.2

$$^8P_5 = \frac{8!}{~~5!~~\,~~3!~~\,3!} = ~~8!~~\; 8 \times 7 \times 6 \times 5 \times 4$$

First employee can be given 8 possible options.

2nd employee ——"—— 7 —"——

3rd ——"—— 6 —"——

4th ——"—— 5 —"——

5th ——"—— 4 —"——

∴ Total options $= 8 \times 7 \times 6 \times 5 \times 4$

Q.3

Possibility that driver will be caught speeding is when she is caught in any one or more gates.

i.e. $P(E) = P(E_1) + P(E_2) + P(E_3) + P(E_4)$

... where E_i is event of being caught for overspeeding at Gate i.

Also, E_i is $S_i \cap T_i$ ... where S_i is ~~probabi~~ event of overspeeding ~~getting caught~~ at gate ~~E~~ i

& T_i is event of traffic ~~Lio~~ radar is ON at the time of passage

$\therefore P(E_i) = P(S_i) \cdot P(T_i)$.

$\therefore P(E) = \sum_{i=1}^{4} P(S_i) \cdot P(T_i)$

$= (0.2)(0.4) + (0.1)(0.3) + (0.5)(0.2) + (0.2)(0.3)$

$= 0.27$

$\therefore$ **Answer = 0.27**

Q.4

let E & F be two events. Then $E = EF \cup EF^c$

$\therefore P(E) = P(EF) + P(EF^c)$ ... (Note F & F^c enforce that EF & EF^c are mutually exclusive. i.e. $P(EF \cap EF^c) = 0$)

$= P(E|F) \cdot P(F) + P(E|F^c) \cdot P(F^c)$

above expression is a weighted average of two conditional probabilities where $P(F)$ & $P(F^c)$ act as ~~a~~ the weights & $P(F) + P(F^c) = 1$.

Q.5

This problem is similar to the one of putting n balls in r baskets such that no basket is empty.

$$\text{Answer} = \binom{n-1}{r-1} = \binom{15-1}{7-1} = \binom{14}{6} = {}^{14}C_6$$

Q.6

D → event that patient has disease.
E → event that lab test is possitive.

$$P(D|E) = \frac{P(DE)}{P(E)} = \frac{P(E|D) \cdot P(D)}{P(E|D) \cdot P(D) + P(E|D^c) \cdot P(D^c)}$$

Based on given data:

$P(D) = 0.005$, $P(D^c) = 0.995$, $P(E|D) = 0.95$

$P(E|D^c) = 0.01$

$$\therefore P(D|E) = \frac{(0.95)(0.005)}{(0.95)(0.005) + (0.01)(0.995)} = 0.323$$

$\therefore$ $P(D|E) = 0.323$ ANSWER