In-Class Test #1 - Module PDE - SOLUTIONS

Engineering Mathematics For Advanced Studies

IIT Dharwad
Autumn 2019

Time - 20 minutes (17th Oct. 2019)
Maximum score - 10
Rule for absentee - Minimum 30% penalty, discuss reasons absense in person to get a chance for re-test.
Note:

1. Please Circle the appropriate option in case of objective questions.
2. For subjective one please use space on the backside of the page

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1. What is the correct name for this equation: $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$ for a function u defined such that $u = u(x, y)$ (mark 1)
   (a) Laplace equation
   (b) Wave equation
   (c) Heat Equation
   (d) Fourier equation
   (e) Poiseuille equation
   (f) Poisson equation $\Longleftarrow$**Answer**
   (g) Hyberbolic Equation

2. We saw the Wave equation solution which started with assumption $u(x, t) = F(x)G(t)$. Which of the following is/are appropriate: (mark 1)
   (a) It is separating functions of x and t $\Longleftarrow$**Correct**
   (b) Differential forms lead to the same order of differential equations for $F(x)$ and $G(t)$ $\Longleftarrow$**Correct**
   (c) $F(x)$ is a sine series and $G(t)$ is cosine series with stipulated phase lag between x and t determined by some constant $\Longleftarrow$**Incorrect**

3. In the derivation shown in the class we had $F" - kF = 0$ and $\ddot{G} - c^2kG = 0$. These two are two completely independent PDEs i.e. without any interdependence (True /False) (mark 1)
   False________

   Why? ____ (mark 1)
   Parameter 'k' is connecting those two solutions together. Hence, those are not independent solutions.

4. Music instrument are tuned typically to get the correct sound. That "tuning" is done by adjusting the tenstion in the string. Which of the variable above is introducing that effect of the tension in above mathematical model of the vibrating string? (mark 2)
   (a) F
   (b) c $\Longleftarrow$**Answer**
   (c) k
   (d) G

5. D'Alembert's solution uses which of the following substitutions: (mark 1)
   (a) $z = u - t$, $v = u + t$
   (b) $u_x = u_z + cu_t$, $u_t = u_x$
   (c) $v = x + ct$, $z = x - ct$ $\Longleftarrow$**Answer**

(d) $u_x = u_z$, $u_t = u_x$

(e) None of above

6. Solve to find general solutions:

(a) $u_{xy} = u_x$ (mark 1)

Answer:

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial u}{\partial x} \quad \Rightarrow \quad \frac{\partial^2 u}{\partial y \partial x} = \frac{\partial u}{\partial x} \quad \Rightarrow \quad \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial u}{\partial x}$$

say $g_1(x, y) = \frac{\partial u}{\partial x}$

$$\frac{\partial g_1(x, y)}{\partial y} = g_1(x, y) \quad \Rightarrow \quad g_1(x, y) = h_1(x) \cdot e^y \quad \Rightarrow \quad \frac{\partial u}{\partial x} = h_1(x) \cdot e^y$$

$$\therefore u = e^y \cdot \int h_1(x) \partial x + h(y) = e^y \cdot h_2(x) + h(y)$$

(b) $u_y = 2xyu$ (mark 1)

Answer:

$$\frac{\partial u}{\partial y} = 2xyu \quad \Longrightarrow \quad \int \frac{\partial u}{u} = 2x \int y \partial y \quad \Longrightarrow \quad ln(u) = 2x(\frac{1}{2}y^2 + h_1(x)) = xy^2 + h(x)$$

$$u = e^{xy^2 + h(x)}$$

or equivalently,

$$u = g(x) \cdot e^{xy^2}$$

7. Identify type of the PDE (Heat/Wave/Laplace/Fourier ... and so on)
$2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} - 3\frac{\partial u}{\partial y} + x = 0$ (mark 1)

It is a second order differential equation for $u = u(x, y)$.
Comparing with canonical form $au_{xx} + bu_{xy} + cu_{yy} = f(u, u_x, u_y, x, y)$ we get $a = 2, \quad b = -1, \quad c = 1 \quad \Longrightarrow b^2 - 4ac = (-1)^2 - 4(2)(1) = -7 < 0 \Longrightarrow$ ELLIPTIC equation