IIT - DHARWAD, FALL 2019

Engineering Maths For Adv. Studies

Solutions – Final Exam PDE Module

Q.1

For 2D problem of 2^{nd} order linear ODE we will adopt easier approach of 'b^2-4ac' assessment. Note that we ensure that PDE is represented so that 'a' is positive. in $a u_{xx} + b u_{xy} + c u_{yy} = f(u_x, u_y, u, x, y)$.

(i) $u_{xx} + 2u_{xy} - 3u_{yy} = 0$

$a = 1, \quad b = 2, \quad c = -3$

$b^2 - 4ac = (2)^2 - 4(1)(-3) = 16$

$b^2 - 4ac > 0 \Longrightarrow$ Hyperbolic equation.

(ii) $-u_{xx} + u_{yy} + u = 0 \Longrightarrow$

$a > 0$ is one precaution. So we reexpress given PDE

$u_{xx} - u_{yy} = u \cancel{\neq 0}$

$a = 1, \quad b = 0, \quad c = -1$

$b^2 - 4ac = (0)^2 - 4(1)(-1) = 4 > 0 \Rightarrow$ Hyperbolic Equation.

(iii) $4u_{xx} + u_{zz} + 4u_{xz} + u_x + u_z = 3x.$

$a = 4, \quad b = 4, \quad c = 1$

$b^2 - 4ac = (4)^2 - 4(4)(1) = 0$

$b^2 - 4ac = 0 \Longrightarrow$ Parabolic Equation.

Q.2

Given PDEs are in 3 dimensions. We ensure that those are 2^{nd} order PDEs. (linear!)

Here we use more structured & formal approach of Hessian matrix of coefficients.

In case of 3 dimensions,

$$A u_{xx} + B u_{yy} + C u_{zz} + 2D u_{xy} + 2E u_{\cancel{x}z} + 2F u_{\cancel{x}yz} = \cancel{f} = f(u_x, u_y, u_z, u, x, y, z)$$

(please note '2' in mixed double derivative terms)

Hessian matrix $\Rightarrow H = \begin{bmatrix} A & D & E \\ D & B & F \\ E & F & C \end{bmatrix}$

Analyze Hessian matrix eigen values. (Applicable for any dim $n>0$)

1. All eigen values have same sign (and are non-zero) $\Rightarrow$ Elliptic PDE
2. At least one of the eigen value is zero $\Rightarrow$ Parabolic PDE
3. No eigen value zero and all have same sign Except for precisely one of them $\rightarrow$ Hyperbolic PDE
4. No eigen value zero and ~~all~~ two or more of those have sign different than the rest of eigen values $\Longrightarrow$ Ultra Hyperbolic PDE (rarely these occur in real life problems)

Q.2

Note: (1) that when $n=3$, i.e. $u=u(x,y,z)$ we only have 3 types of PDEs → Elliptic, Parabolic, Hyperbolic.

(2) We can also leverage linear algebra learning that

- signs of pivots & signs of eigen values sum totol to same number.
- positive-definite matrix would have all the upper left submatrices with positive determinant.
- determinant of a matrix is equal to the product of its eigen values.

  Hence, if any one eigen value is zero ⇒ determinant $=0$

(3) So, in case of $n=3$ i.e. $u=u(x,y,z)$ & $A>0$ we can say,

if $|H|=0 \Rightarrow$ parabolic

if $|H|>0$ && $|H_2|>0$ && $|H_1|=A>0$
$\Longrightarrow$ Elliptic.

Else $\Longrightarrow$ Hyperbolic.

Actual answers for 2^{nd} order PDE for $u=u(x,y,z)$ starts on next page.

$\boxed{Q.2}$

(i) $u_{yy} + \sin(y) + 6u_{zz} = u_x$.

$$u_{yy} + 6u_{zz} = f(z,x,y,u,u_x,u_y)$$

$$\left(Au_{yy} + Bu_{zz} + Cu_{xx} + 2\overset{u_z}{D}u_{yz} + 2Eu_{yx} + 2Fu_{zx} = f(u_x,u_y,u_z,u,x,y,z)\right)$$

$\therefore$ $A = 1$, $B = 6$, $C = D = E = F = 0$.

$\because$ (notice we use $u = u(y,z,x)$ instead of typical $u = u(x,y,z)$)

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Longrightarrow |H| = 0 \Longrightarrow \text{Parabolic PDE}$$

(ii) $u_{xx} + u_{zz} + e^x = 4u_{yz} + u_x$

$u = u(x,y,z)$

$u_{xx} + u_{zz} - 4u_{yz} = u_x + e^x$ i.e. $f(u_x,u_y,u_z,u,x,y,z)$

$\therefore$ $A = 1$, $B = 0$, $C = 1$, $D = 0$, $E = 0$, $F = -2$

(Compared to
$$Au_{xx} + Bu_{yy} + Cu_{zz} + 2Du_{xy} + 2Eu_{xz} + 2Fu_{yz} = f(u_x,u_y,u_z,u,x,y,z))$$

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & -2 & 1 \end{bmatrix} \Rightarrow |H| = -4 \Longrightarrow \underline{\text{Hyperbolic}}$$

$$|H_2| = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$$|H_1| = 0$$

<u>Aside</u>

Characteristic eqn. to get eigen values $\lambda_i \Longrightarrow |H - \lambda I| = 0$

$$\begin{vmatrix} (1-\lambda) & 0 & 0 \\ 0 & -\lambda & -2 \\ 0 & -2 & (1-\lambda) \end{vmatrix} = 0 \Rightarrow (1-\lambda)\left[-\lambda(1-\lambda) - 4\right] = 0 \Rightarrow \text{one of eigen value } \lambda = 1$$

$\Downarrow$

Hyperbolic $\Leftarrow$ Two of 3 eigen values are +ve & other on is -ve $\Leftarrow$ other two should have product -4 as $|H| = -4$

Q.3

(a) True

(b) True $\quad \nabla^2 u = 0 \Rightarrow u_{xx} + u_{yy} = 0 \Rightarrow u_{\mu\mu} + u_{\eta\eta} = 0$
(see assignment problem)

(c) False.

$X = ax$, $Y = ay$ say $\mu = ax$ & $\eta = ay$

$\therefore u_x = u_\mu \cdot \mu_x = a u_\mu$ note $x = x(\mu)$ & $y = y(\eta)$

$u_y = u_\eta \cdot \eta_y = a u_\eta$

$\therefore u_{xx} = a^2 u_{\mu\mu}$ & $u_{yy} = a^2 u_{\eta\eta}$

$\therefore \nabla^2 u = f(x,y) \Rightarrow a^2 \cdot (u_{\mu\mu} + u_{\eta\eta}) = f(x,y).$

$\Rightarrow \nabla^2 u \Big|_{\mu - \eta} = \frac{1}{a^2} f(x,y) \neq f(x,y).$

Q.4

Given. $\quad u = u(t,y)$

(a) & $\quad \frac{\partial u}{\partial t} = -2u \Longrightarrow$ Governing equation

(b) $\int \frac{\partial u}{u} = -2 \int \partial t$

$\ln(u) = -2[t + h_1(y)] = -2t + h_2(y)$

$\therefore u = e^{-2t + h_2(y)} = h_3(y) \cdot e^{-2t}$

$\therefore u = h(y) \cdot e^{-2t}$

Q.4

Given $u = u(t, y)$ → displacement.

Hence, acceleration $\frac{\partial^2 u}{\partial t^2}$.

∴ Governing Eqn.

$$\frac{\partial^2 u}{\partial t^2} = -2u$$

i.e.

$$u_{tt} = -2u$$

Note that problem involves derivatives with only t as no y term is involved in above PDE. so we can solve it like ODE. (Refer example 2 & 3 before problem set 11.1 in Kreyszig. 8th Edn.)

$$u'' = -2u.$$

$$u'' + 2u = 0$$

assuming solution of the form Ce^{st}

$u' = s.u$ & $u'' = s^2 u$.

∴ $(s^2 + 2)u = 0$ .. ($\because$ $u'' + 2u = 0$)

∴ $s = \pm i\sqrt{2}$

∴ $u = h_1(y)\, e^{+i\sqrt{2}t} + h_2(y)\, e^{-i\sqrt{2}t}$.

— Note that coeffiecient are f^n of (y).

$\boxed{Q.5}$

[Refer to the problem 8 in problem set 11.9 in Kreyszig edition 8. (~~Problem~~ Text before the problem 8. ($\alpha(\theta) = 10\cos^2(\theta)$)) lays out the procedure]

~~From~~ Formula~~r~~ for polar coordinate system ~~heat diffu~~ Laplace equation ($\nabla^2 E = 0$ & semicircular plate).

$$u(r,\theta) = a_0 + \sum_{n=1}^{\infty}\left[a_n\left(\frac{r}{R}\right)^n \cos(n\theta) + b_n\left(\frac{r}{R}\right)^n \sin(n\theta)\right] \quad \text{—— } 5a$$

— (eqn (6) on pg. 628, under problem set 11.9 in Kreyszig.)

Given B.C. $u(\theta) = u(R,\theta) = 4\cos^2\theta = 2(2\cos^2\theta)$

$$= 2(\cos 2\theta + 1)$$

... (using trignometric formula provided in question paper)

Boundary Cond^n:

$$\therefore u(R,\theta) = 2 + 2\cos 2\theta \quad \text{—— } 5b$$

5a coefficients a_0, a_i & b_i ($i = 1, 2 \ldots \infty$) have to obey 5b when $r = R$.

5a Reduces to following eqn. for $r = R$.

$$u(R,\theta) = a_0 + \sum_{n=1}^{\infty}\left[a_n\cos(n\theta) + b_n\sin(n\theta)\right] \quad \text{—— } 5c$$

Comparing 5b & 5c

$a_0 = 2$, $a_1 = 0$, $a_2 = 2$, & $a_i = 0$ for $i > 2$

$b_j = 0$ for all $j > 0$.

5a becomes → $\boxed{u(r,\theta) = 2 + 2\left(\frac{r}{R}\right)^2 \cos(2\theta)}$

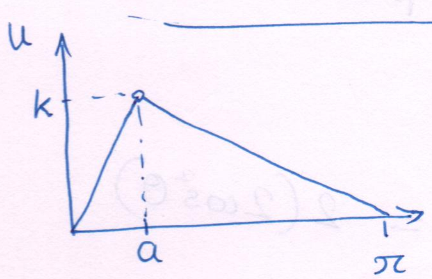
$\boxed{Q.6}$

Boundary condition in Q.5, prescribes actual value 'u' of the solution at the given boundary $\Rightarrow$ Dirichlet BC

(Refer. section 11·5 steady state 2D heat flow) in Kreyszig 8th Ed.

$\boxed{Q.7}$

(Reference $\rightarrow$ Kreyszig 8th Ed., problem set 11·3, & solved problem example 1 on previous page in book)



ⓐ The given problem is a string vibration problem. Hence, 1-D wave equation will govern it:

$$u_{tt} = c^2 u_{xx}$$

ⓑ Kreyszig, section 11·3 : separation of variables ...., pg 591 in 8th Ed.

eqn. (12) gives final solution for above wave eqn.

$$u(x,t) = \sum_{n=1}^{\infty} \left(B_n \cos(\lambda_n t) + B^* \sin(\lambda_n t) \right) \sin\left(\frac{n\pi x}{L}\right)$$

where $\lambda_n = \dfrac{cn\pi}{L}$ & B_n & B^* coefficients are

$$B_n = \frac{2}{L} \int_0^L f(x) \cdot \sin\left(\frac{n\pi x}{L}\right) dx \quad \cdots n = 1, 2, 3 \ldots \infty$$

$f(x) \rightarrow$ initial displacement.

$$B_n^* = \frac{2}{cn\pi} \int_0^L g(x) \cdot \sin\left(\frac{n\pi x}{L}\right) dx \quad \cdots n = 1, 2, 3 \cdots \infty$$

$g(x) \rightarrow$ initial velocity

ⓒ Simplify above expression noting that $c^2 = 1$, $L = \pi$ for the given problem.

Hence,

$$\lambda_n = \frac{cn\pi}{L} = \frac{(1)\, n \cdot \pi}{\pi} = n$$

$$\& \quad \frac{n\pi x}{L} = nx$$

Simplified: I

$$\therefore u(x,t) = \sum_{n=1}^{\infty} \left(B_n \cos(nt) + B^* \sin(nt) \right) \sin(nx)$$

where

$$\left.\begin{aligned} B_n &= \frac{2}{\pi} \int_0^{\pi} f(x) \cdot \sin(nx)\, dx \\ B_n^* &= \frac{2}{n\pi} \int_0^{\pi} g(x) \cdot \sin(nx)\, dx \end{aligned}\right\} n = 1, 2, \cdots \infty$$

ⓓ Given that initial velocity is zero $\Rightarrow g(x) = 0$.

$$g(x) = 0$$

Hence

$$B_n^* = 0 \qquad \text{for} \quad n = 1, 2, \cdots \infty.$$

Simplified Eqn.: II

$$u(x,t) = \sum_{n=1}^{\infty} B_n \cos(nt) \sin(nx)$$

where, $$B_n = \frac{2}{E\pi} \int_0^{\pi} f(x) \sin(nx)\, dx$$

Q.7 continued...

(e) $$f(x) = \frac{kx}{a} \quad \ldots \text{ if } 0 \leq x \leq a$$

$$= \left(\frac{\pi - x}{\pi - a}\right) k \quad \ldots \text{ if } a < x \leq \pi$$

(f) Only remaining exercise is to evaluate integrals for estimation of B_n coefficients noting the function $f(x)$ is given differently for $x \leq a$ & for $x > a$.

$$B_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cdot \sin(nx)\, dx$$

$$= \frac{2}{\pi} \left[\int_0^a \frac{kx}{a} \sin(nx)\, dx + \int_a^{\pi} \frac{(\pi - x)k}{(\pi - a)} \sin(nx)\, dx \right]$$

$$= \frac{2k}{\pi a} \int_0^a x \cdot \sin(nx)\, dx + \frac{2\pi k}{\pi(\pi - a)} \int_a^{\pi} \sin(nx)\, dx$$

$$\cancel{+} - \frac{2k}{\pi(\pi - a)} \int_a^{\pi} x \sin(nx)\, dx$$

$$= \frac{2k}{\pi a} I_1 + \frac{2\cancel{\pi}k}{(\pi - a)} I_2 + \frac{2k}{\pi(\pi - a)} I_3 .$$

where I_1, I_2 & I_3 are integrals given as ~~below~~: follows:

Q.7 continued

$$I_1 = \int_0^a x \cdot \sin(nx)\,dx$$

$$I_2 = \int_a^\pi \sin(nx)\,dx = \left.\frac{-\cos(nx)}{n}\right|_a^\pi = \frac{\cos(an)}{n} - \frac{\cos(\pi n)}{n}$$

$$= \frac{1}{n}\left[\cos(an) - (-1)^n\right]$$

$$I_3 = \int_a^\pi x \cdot \sin(nx)\,dx$$

(g) Now we need to evaluate $\int x \cdot \sin(nx)\,dx$.

we will use $\int uv' = uv - \int u'v$

Here $v' = \sin(nx) \Rightarrow v = \left(\frac{-\cos(nx)}{n}\right)$.

$$\therefore \int x \cdot \sin(nx)\,dx = \int_{s_1}^{s_2} x \cdot \frac{d}{dx}\left(\frac{-\cos(nx)}{n}\right)dx$$

$$= \left.\frac{-x\cos(nx)}{n}\right|_{s_1}^{s_2} - \int_{s_1}^{s_2} \frac{-\cos(nx)}{n}\,dx$$

$$= \left.\frac{-x\cos(nx)}{n}\right|_{s_1}^{s_2} + \left.\frac{1}{n^2}(\sin(nx))\right|_{s_1}^{s_2}$$

$$= \frac{1}{n^2}\left[n s_1 \cos(s_1 n) - n s_2 \cos(s_2 n) + \sin(s_2 n) - \sin(s_1 n)\right]$$

$$I_1 = \int_0^a x \cdot \sin(nx) = \frac{1}{n^2}\Big[\sin(an) - an\cos(an)\Big]$$

$$I_3 = \int_a^\pi x \cdot \sin(nx)$$

$$= \frac{1}{n^2}\Big[an\cos(an) - \pi n\cos(\pi n) + \underbrace{\cancel{\sin(\pi n)}}_{=0} - \sin(an)\Big]$$

$$= \frac{1}{n^2}\Big[an\cos(an) - \pi n(-1)^n - \sin(an)\Big]$$

Q.8

$$x u_{xx} + (x-y) u_{xy} + y u_{yy} = 0 \qquad x, y > 0$$

ⓐ 2 Dimensional, linear, second-order PDE

$a = x$, $b = (x-y)$, $c = y$ ...

(Note :- for $b^2 - 4ac$ approach we need to ensure that $a > 0$ which is ensured in this problem).

$\therefore$ $b^2 - 4ac > 0 \Rightarrow$ Hyperbolic PDE.

$\therefore$ $(x-y)^2 - 4x \cdot y > 0 \Longrightarrow$ Necessary condition for PDE being hyperbolic

ⓑ Suggested transformation $\rightarrow$ $\mu = x - y$

$\eta = xy$

we need to change variables to express u_{xx}, u_{yy}, u_{xy} in terms of $u_{\mu\mu}$, $u_{\eta\eta}$, u_μ, u_η, u.

Given, transformation shows that $x = x(\mu, \eta)$

$y = y(\mu, \eta)$.

$$\therefore \frac{\partial u}{\partial x} = \cancel{u} \; u_x = u_\mu \cdot \frac{\partial \mu}{\partial x} + u_\eta \cdot \frac{\partial \eta}{\partial x}$$

similarly

$$\frac{\partial u}{\partial y} = u_y = u_\mu \cdot \frac{\partial \mu}{\partial y} + u_\eta \cdot \frac{\partial \eta}{\partial y}$$

Lets evaluate μ_x, μ_y, η_x & η_y

$$\mu_x = \frac{\partial \mu}{\partial x} = 1, \quad \mu_y = \frac{\partial \mu_y}{\partial y} = -1, \quad \eta_x = \frac{\partial \eta}{\partial x} = y, \quad \eta_y = \frac{\partial \eta}{\partial y} = x$$

$$\therefore \mu_x = 1 \quad , \quad \mu_y = -1 \quad , \quad \eta_x = y \quad , \quad \eta_y = x$$

$$\therefore u_x = u_\mu \cdot (1) + u_\eta (y) = u_\mu + u_\eta y$$

$$u_y = u_\mu (-1) + u_\eta (x) = -u_\mu + u_\eta \cdot x$$

$$\therefore u_{xx} = (u_\mu + y u_\eta)_x$$

$$= u_{\mu\mu} \cdot \mu_x + u_{\mu\eta} \cdot \eta_x + y \left(u_{\eta\mu} \cdot \mu_x + u_{\eta\eta} \cdot \eta_x \right)$$

$$= u_{\mu\mu} + u_{\mu\eta} \cdot y + y u_{\eta\mu} + y^2 u_{\eta\eta}$$

$$u_{xx} = u_{\mu\mu} + 2y u_{\mu\eta} + y^2 u_{\eta\eta}$$

$$u_{yy} = (u_y)_y = (-u_\mu + u_\eta x)_y$$

$$= -(u_{\mu\mu} \cdot \mu_y + u_{\mu\eta} \eta_y) + x \left(u_{\eta\mu} \cdot \mu_y + u_{\eta\eta} \cdot \eta_y \right)$$

$$= u_{\mu\mu} - x u_{\mu\eta} - x u_{\eta\mu} + x^2 u_{\eta\eta}$$

$$u_{yy} = u_{\mu\mu} - 2x u_{\mu\eta} + x^2 u_{\eta\eta}$$

$$u_{xy} = (u_\mu + u_\eta y)_y$$

$$= u_{\mu\mu} \mu_y + u_{\mu\eta} \eta_y + y \left(u_{\eta\mu} \cdot \mu_y + u_{\eta\eta} \eta_y \right) + u_\eta \cdot (1)$$

$$= -u_{\mu\mu} + x u_{\mu\eta} - y u_{\eta\mu} + xy u_{\eta\eta} + u_\eta$$

$$u_{xy} = -u_{\mu\mu} + (x-y) u_{\mu\eta} + xy u_{\eta\eta} + u_\eta$$

Putting u_{xx}, u_{yy}, u_{xy} back in given PDE

$$x u_{xx} + (x-y) u_{xy} + y u_{yy} = 0$$

$$x\left(u_{\mu\mu} + 2y u_{\mu\eta} + y^2 u_{\eta\eta}\right) + (x-y)\left(-u_{\mu\mu} + (x-y) u_{\mu\eta} + xy u_{\eta\eta} + u_\eta\right)$$

$$+ y\left(u_{\mu\mu} - 2x u_{\mu\eta} + x^2 u_{\eta\eta}\right) = 0$$

Note → Due to typo in question ~~last~~ +ve sign occured for $y u_{yy}$ term in the last. It should have been +ve to arrive at Canonical form in terms of μ & η. We don't arrive at that here. However, Full marks will be awarded any student who reached above stage.

$$2x u_{\mu\mu} + (x-y)^2 u_{\mu\eta} + 2x^2 y u_{\eta\eta} + (x-y) u_\eta = 0$$

$$2x u_{\mu\mu} + \mu^2 u_{\mu\eta} + 2x\eta u_{\eta\eta} + \mu u_\eta = 0$$

⟶ Not a canonical form.

<u>For additional in.</u>

With correct PDE $x u_{xx} + (x-y) u_{xy} - y u_{yy} = 0$

we would get, after substitution

$$(x^2 + 2xy + y^2) u_{\mu\eta} + (x-y) u_\eta = 0$$

$$\left((x-y)^2 + 4xy\right) u_{\mu\eta} + (x-y) u_\eta = 0$$

since $x - y = \mu$ & $xy = \eta$,

$$(\mu^2 + 4\eta) u_{\mu\eta} + \mu u_\eta = 0 \Rightarrow \text{Canonical Form.}$$

Q.9

ⓐ False. $\nabla^2 u = 0$ is Laplace eqn. Canonical form

ⓑ True

$\nabla^2 u = u_t$ is heat equation.

Q.10

$\rho_{correct} = 4\rho_{erroneous}$ say $\rho_{new} = 4\rho_{old}$

Refer section 11.5: 'Heat equation: Solution by Fourier Series' in Kreyszig, (pg. 602 in 8th Ed.)

$$u_n(x,t) = B_n \sin\left(\frac{n\pi x}{L}\right) e^{-\lambda_n^2 t}$$

& $$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n\pi x}{L}\right) \cdot e^{-\lambda_n^2 t}$$

where $$B_n = \frac{2}{L}\int_0^L f(x) \cdot \sin\left(\frac{n\pi x}{L}\right) dx.$$

Note that B_n is not dependent on ρ (density) as none of the variables are related to material property. Same argument applies for $\sin\left(\frac{n\pi x}{L}\right)$ term. Hence, for the sake of comparison of ρ_{new} & ρ_{old} we treat those as constant. Let's ~~same~~ say t_0 is the specified time at which ~~res~~ value of u at point x_0 is to be estimated to reach value T.

$\boxed{Q.10}$

$$T = \sum_{n=1}^{\infty} c_n \, e^{-\lambda_{old}^2 t_{old}}$$

However $\quad {}_n\lambda_{old} = \frac{c n\pi}{L} = \sqrt{\frac{K}{\sigma \rho_{old}}} \cdot \frac{n\pi}{L} \quad \cdots \left(c^2 = \frac{K}{\sigma\rho}\right)$

$${}_n\lambda_{new} = \sqrt{\frac{K}{\sigma \rho_{new}}} \cdot \frac{n\pi}{L} = \sqrt{\frac{K}{4\sigma\rho_{old}}} \cdot \frac{n\pi}{L} = \frac{\lambda_{old}}{2}$$

$\therefore \; {}_n\lambda_{old}^2 = 4\, {}_n\lambda_{new}^2 \qquad \cdots (\because \rho_{new} = 4\rho_{old})$

$$\therefore \quad T = \sum_{n=1}^{\infty} c_n \cdot e^{-{}_n\lambda_{old}^2 \cdot t_{old}} = \sum_{n=1}^{\infty} c_n \cdot e^{-4\,{}_n\lambda_{new}^2 \, t_{old}}$$

Also

$$T = \sum_{n=1}^{\infty} c_n \cdot e^{-{}_n\lambda_{new}^2 \cdot t_{new}} \qquad \#$$

subtraction gives:

$$\therefore \quad \sum c_n \left(e^{-{}_n\lambda_{new}^2 \cdot t_{new}} - e^{-4\,{}_n\lambda_{\cancel{old}\, new}^2 \cdot t_{\cancel{new}\, old}} \right) = 0$$

$$\therefore \quad \sum c_n \left(A_n^{t_{new}} - A_n^{-4 t_{old}} \right) = 0$$

$$\text{where } A_n = e^{-{}_n\lambda_{new}^2}$$

$\therefore$ We land-up on an implicit equation. relating t_{new} with t_{old}. However, explicit expression is difficult if not impossible!

$\boxed{Q.11}$

a) TRUE - Similarity solution approach can solve non-linear PDE.

b) TRUE.