# PDE-Final Exam

**Engineering Mathematics for Advanced Studies**

IIT Dharwad Autumn 2019

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Date - Thursday 7th Nov. 2019
Time - 2 Hours (8:30am -10:30am)
Maximum score - 60; Minimum score - 0
Rule for absentee - Minimum 30% penalty; discuss reasons absense in person to get a chance for re-test.

1. This is an open book exam; most of the answers will require referring to the book.
2. Worked out solutions are must for most of the problems
3. If not stated explicitly, treat dependent variable $u = u(x, y, z, t)$ where $x, y$ and $z$ are spatial variables and $t$ is time variable
4. Some possibly useful identities:
   $cos(2\theta) = cos^2(\theta) - sin^2(\theta) = 1 - 2sin^2(\theta) = 2cos^2(\theta) - 1$
5. Please ensure to write question number in a box as a heading to the upcoming answer on suppliments e.g.

$\boxed{\text{Question 1}}$

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1. Classify following equations as Elliptical/Parabolic/Hyperbolic equation. Write NA in answer script if none of these are appropriate for the equation? Show your work. (marks 6)

| No. | Description of $u$ | Equation |
|---|---|---|
| i. | $u = u(x, y)$ | $u_{xx} + 2u_{xy} - 3u_{yy} = 0$ |
| ii. | $u = u(x, y)$ | $-u_{xx} + u_{yy} + u = 0$ |
| iii. | $u = u(x, z)$ | $4\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} + 4\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial z} = 3x$ |

2. Classify following equations as Elliptical/Parabolic/Hyperbolic equation. Write NA in answerscript if none of these are appropriate for the equation? Show your work. (marks 6)

| No. | Description of $u$ | Equation |
|---|---|---|
| i. | $u = u(x, y, z)$ | $\frac{\partial^2 u}{\partial y^2} + sin(y) + 6\frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial x}$ |
| ii. | $u = u(x, y, z)$ | $u_{xx} + u_{zz} + e^x = 4u_{yz} + u_x$ |

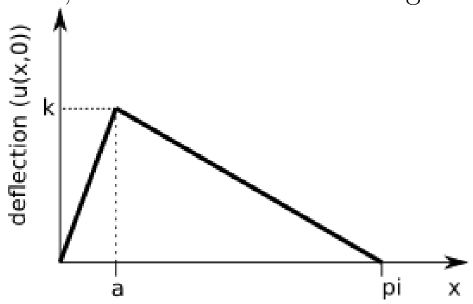
3. State True or False -
   (a) It is possible that the same partial differential equation can have two different solutions (marks 2)
   (b) Laplacian equation is invariant under rotation transformation of the coordinate system (marks 2)
   (c) Poisson equation is invariant under scaling transformation of the coordinate system (marks 2)

4. It is found that the acceleration of certain particles being studied is always in the opposite direction considering its displacement $u$ and is 2 times the displacement $u$. Particles moves only along one direction say $x$ (to be explicit - both +ve and -ve x-directions). This displacement $u$ depends on initial $y$ coordinate of the particle and the time $t$.
   (a) Can you write governing equation for the above? (marks 2)
   (b) Can you get expression for $u$ ? (marks 4)

5. If E is electrostatic potential, for a region free of charges one can write $\nabla^2 E = 0$. Now consider a semicircular plate of radius $R = 1$. Find the distribution of electrostatic potential $E(r, \theta)$ if boundary condition is given by $E(\theta) = 4cos^2(\theta)$. (Refering to book to identify process/formula(e) is recommended) (marks 8)

6. What is the type of boundary condition mentioned in above problem 5 ( Newton/ Dirichelet/ Neumann/ Mixed)? (marks 2)

7. Find deflection u(x,t) of a string of length $L = \pi$ when $c^2 = 1$, the initial velocity is zero, and initial deflection is given by following diagram: (marks 8)




8. Find canonical form (i.e.) standard form in which Laplace/Heat/Wave equations are generally expressed for the following using the guidelines:

$$xu_{xx} + (x - y)u_{xy} + yu_{yy} = 0 \qquad x > 0, y > 0$$

(a) Are there any conditions on $(x, y)$ under which above equation will be hyperbolic? (marks 2)

(b) Consider following transformation:

$$\mu = x - y \qquad \eta = xy$$

Express given PDE in terms of $u = u(\mu, \eta)$ using appropriate change of variables. This form is expected to be the canonical form. (marks 6)

9. True or False:

(a) Following is Laplace equation for $u = u(r, \theta, z)$: (marks 2)

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} + u_{zz} = rsin(\theta)$$

(b) Following is Heat equation for $u = u(r, \theta, \phi)$: (marks 2)

$$\nabla^2 u = \frac{\partial u}{\partial t}$$

10. In the heat diffusion scenario (heat equation) the certain scenario required time $t_1$ for a certain point $x$ in the domain to reach temperature $T$ as per the calculation of a new student. However, advisor finds that the value of density used by the student was one fourth of the correct density value. What will be the correct time $t_2$ estimate in terms of the earlier incorrect estimate of time $t_1$ for the same point $x$ to reach same temperature $T$ ? (marks 4)

11. Regarding solutions by similarity variables state True or False:

(a) Similarity variable method can solve non-linear PDEs (marks 1)

(b) Identifying stretching transformation under which PDE is invariant is a part of the process (marks 1)