PDE-Assignment #1

Engineering Mathematics for Advanced Studies
IIT Dharwad
Autumn 2019

Submission - Monday 28th Oct. 2019 5:30pm
Late penalty - 1 day late* 30%, 100% for more than a day (*starts from 5:31pm, 28th Oct. 2019!)
Total marks - 20

1. Wave equation in its standard form is given by $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. However in certain problems there may be an extra force term that will cause difficulty in separation of variables. e.g. consider $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + Ax$ for $0 < x < L$ and $t > 0$ with boundary conditions $u(0,t) = u(L,t) = 0$ for $t \geq 0$ and initial contitions $u(x,0) = 0$, $\frac{\partial u}{\partial t}(x,0) = 1$ for $0 < x < L$?

   (a) Can you think of a typical physics problems that would result in a force term used in this problem? (marks 1)

   (b) Verify that separation of variable will not work for the above problem. (hint: substitute $u(x,t) = F(x)G(t)$ and attempt to separate x and t terms) (marks 2)

   (c) Now transform the problem using the following substitution for $u(x,t)$:

   $$u(x,t) = y(x,t) + \psi(x)$$

   write the Wave equation for above form of $u(x,t)$ (marks 2)

   (d) Can there be appropriate choice of $\psi(x)$ that can bring it to the form: $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$? Please do the necessary integration (integrate twice) and get the general form (marks 2)

   (e) Find the two constants in expression for $\psi(x)$ using boundary conditions given above. Target $u(0,t) = y(0,t)$ and $u(L,t) = y(L,t)$ while getting the constants to make life easier ☺ (marks 2)

   (f) Sum up the transformed problem by listing:

      i. Differential equation in terms of $y(x,t)$ (marks 1)
      ii. Two boundary conditions (marks 1)
      iii. Two initial conditions (marks 1)

2. Assuming axial symmetry the wave equation in polar coordinates is (marks 6)

   $$\frac{\partial^2 z}{\partial t^2} = c^2 \left(\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r}\right)$$

   Initial position $z(r,0) = f(r)$ and initial velocity $\frac{\partial z}{\partial t}(r,0) = g(r)$
   Please find two separate ODEs for above problem following the same procedure used in the class for wave equation in cartesian form.

3. Identify type of the following PDE (elliptic/parabolic/hyperbolic):
   $\frac{\partial^2 u}{\partial x^2} - 3\frac{\partial^2 u}{\partial y^2} + 2\frac{\partial u}{\partial y} + u - y = constant$ (marks 2)