

Engineering Mathematics for Advanced Studies

Autumn 2019
Date – 16th Sept. 2019
Marks – 20

ODE Quiz01
Time – 25 minutes

Rule for absentee - Minimum 30% penalty, discuss reasons absence in person to get a chance for re-test.

Note –

1. Considering the time allotted for the quiz, to be on the safer side, student may want to first go through all questions and identify the ones that would not take much time to answer. Answer these first and then work on the relatively tedious questions.
2. Ensure to clearly write question number before the answer the question.
3. Worked out solutions on supplements are must for some problems.
4. Please ensure to write Question number in a box as a heading to the upcoming answer on supplements e.g.

Question 1

=====

1) Which one of the following is a best match for a Taylor series: (1 mark)

a) $\sum_{n=0}^{\infty} \frac{(\Delta x)^n}{n!} (y^{(n)}(x_0))$

b) $\sum_{n=0}^{\infty} \frac{(x)^n}{n!}$

c) $\sum_{n=0}^{\infty} (x)^n$

d) $\sum_{n=0}^{\infty} \frac{(\Delta x)^{n-1}}{(n-1)!} (y^{(n)}(x_0))$

(note: $y^{(n)}(x_0)$ denotes n'th derivative of the y evaluated at $x = x_0$)

2) An investor puts Rs. 10000 in fixed deposit scheme where interest is compounded almost at every instant at the rate of interest 6% per year for 5 years. Then at the end the 2nd year he invests additional Rs.5000 in the same scheme in the accrued amount.

a) Can you write a single differential equation that models above scenario (please clearly mention the meaning of the variables)? (1 mark)

b) What is the solution to the differential equation and final amount accumulated at the end of 5 years? (1 mark)

3) For given ODE please answer

a) Solve: $\frac{dy}{dx} = 2 - 4y$ with $y(0) = 3$ (1 mark)

b) For above ODE problem, $y(\infty) = ?$ (1 mark)

4) Find complete solution for:

a) $y' - 8y = 2H(t - 3)$ (1 mark)

b) $y' - 8y = e^{2t}$ (1 mark)

5) What are values of M and N in case of proposed solution $y = M\cos(\omega t) + N\sin(\omega t)$ for ODE given by $y' - 5y = \cos(4t) + 2\sin(4t)$? (2 mark)

6) What is R and ϕ in following polar form conversion:

$\cos(2t) + \sin(2t) = R\cos(2t - \phi)$ (2 mark)

7) What is α and G in the $Ge^{-i\alpha} = \frac{1}{re^{i\alpha}}$ where r and α correspond to the polar form

$re^{i\alpha} = \sqrt{3}i + 1$ (2 mark)

8) What are the i) order, ii) degree, iii) Homogeneous/Non-homogeneous, iv) Linear/non-linear attributes corresponding to following two differential equations:

a) $(y'')^2 + (y')^3 - 3y - \cos(x) = 0$ (1 mark)

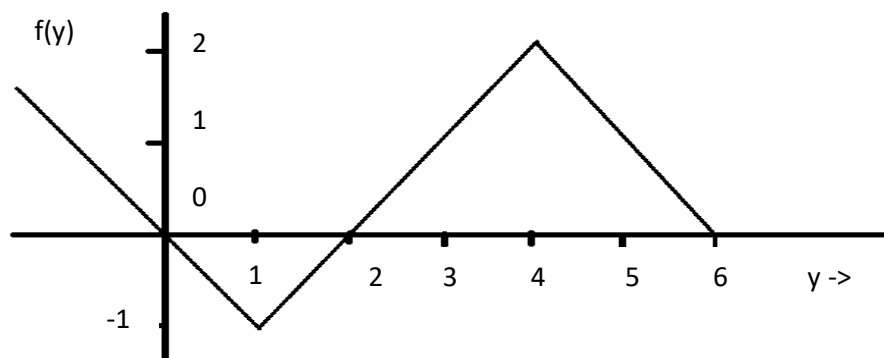
b) $(y'')^2 - 3y'''y^3 = 0$ (1 mark)

9) For the $y' = f(y)$ equation where $f(y)$ is given by following plot, estimate $y(\infty)$ if

a) $y(0) = 1.01$ (1 mark)

b) $y(0) = 2.01$ (1 mark)

c) $y(0) = 5.01$ (1 mark)



10) Does following ODE satisfy exactness condition? $(1 - 2xy^2)dx - (2x^2y)dy = 0$ (2 marks)