ODE-Final Exam

Engineering Mathematics for Advanced Studies
IIT Dharwad
Autumn 2019

Date - Monday 14th Oct. 2019
Time - 2 Hours (5:30pm -7:00pm)
Maximum score - 90; Minimum score -
Rule for absentee - Minimum 30% penalty; discuss reasons absense in person to get a chance for re-test.

1. Worked out solutions are must for some problems.
2. Formulae for standard solutions will be provided on Blackboard.
3. Please ensure to write Question number in a box as a heading to the upcoming answer on supplements e.g. $\boxed{\text{Question 1}}$

One of the key formula for $y' = a \cdot y + q(t)$:
$y(t) = e^{at}y(0) + e^{at}\int_0^t e^{-as}q(s)ds$
One of the key formula for $y' = a(t) \cdot y + q(t)$:
$y(t) = G(0,t).y(0) + \int_0^t G(s,t)q(s)ds$ where,
$G(0,t) = e^{\int_0^t a(s)ds}$ and $G(s,t) = e^{\int_s^t a(u)du}$

Notes provided on blackboard during the exam (appended here post exam):

Surface area of a sphere $= 4\pi r^2$ where r is the radius of the sphere

Volume of a sphere $= \frac{4}{3}\pi r^3$ where r is the radius of the sphere

$q(t)$	y_p
$q(t) = q = constant$	$y_p = \frac{q}{a}(e^{at} - 1)$
$q(t) = H(t)$	$y_p = \frac{1}{a}(e^{at} - 1)$
$q(t) = \delta(t)$	$y_p = e^{at}$
$q(t) = e^{ct}$	$y_p = \frac{e^{ct}}{c-a}$
$q(t) = Re^{i\omega t}$	$y_p = \frac{Re^{i\omega t}}{i\omega - a}$

QUESTIONS

1. Specify Order/Degree/Linear-NonLinear/Homogeneous-Nonhomogeneous parameters for the following set of equations for y(x)? (marks 8)

No.	Equation	Order (N)	Degree (N)	Linear (L) or Non-linear (NL) (L/NL)	Homogeneous (H) / Non-homogeneous (NH) (H/NH)	Autonomous
1	$(y'')^3 + e^y = x$					
2	$y'' + 3yy' + y = 0$					
3	$y'' + (y')^2 + 4y = e^x$					
4.	$y'' + 3yy' + y = sin(y)$					

2. A tank contains 1000 gallons of water in which 200 pounds of salt are dissolved. Fifty Gallons of brine (i.e. salted water) rushes into the tank per minute. Brine has variable total dissoved quantity of salt given by $(1 + cos(t))$pounds and it mixes with the salted water in the tank. The mixture in the tank is made homogenous almost instantaneously by a stirrer and then the homogenous content leave as the same rate i.e. 50 Gallons per minute.
   (a) Please provide a differential equation for the salt content in the tank at any given time t. (marks 6)
   (b) State $a(t)$ and $q(t)$ for this example if standard form is $y' = a(t)y + q(t)$. (marks 2)

3. A moth repellent ball which evaporates and shrinks in radius over a period of time ia found to decrease from 2cm to 1cm in 2 months. If the volume loss is proportional to the area exposed to air, assuming that the spherical shape is maintained throughout its life, can you predict after how many month the diameter will reduce to 0.1cm? (marks 8)

4. Sketch by hand the direction fields for the following: (marks 6)

   (a) $y' = x^2$

   (b) $y' = x + y$

5. Solve: $y' + 9y = 90$ (marks 6)

6. Solve: $y' + y = 8e^{-3t}$ (marks 6)

7. Solve: $y' + y = 3\delta(t-4)$ with $y(0) = 2$ (marks 10)

8. Solve: $z' + z = e^{8it}$ where z is a complex solution. Please provide final answer for z in polar format. (marks 6)

9. Check the exactness of the following equation: $2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0$ (marks 6)

10. Express 3sin(4t) + 4cos(4t) as $Rcos(\omega t - \phi)$ (marks 4)
    $R = ?$ $\phi = ?$

11. If temperature of a cake is 150 Degree C when it leaves oven and is 110 Degree C in 10 minutes later. Assume room temp. to be 20 Degree C.

    (a) State the explicit equation that gives temperature T at any given time t (expressed in minutes). (marks 6)

    (b) After how many minutes will it reach human body temperature 37 Degree C so that healthy individual does not find it hot anymore. (marks 2)

12. Solve :$y" = 2y' + 35y$ (marks 6)

13. Solve :$y" = y' + y$ (marks 8)