Autumn 19, IIT DHARWAD.

course → Engg. Mathematics for Advanced Studies

Module → Numerical Methods.

Solutions to Assignment 02

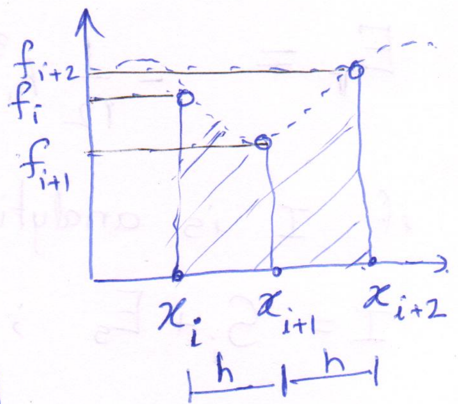
Q.1

(For this question, please refer to Parviz Moin Book, pg-29-30. for details) if $I = \int_{x_j}^{x_{j+1}} f(x)\,dx$

Rectangle Rule (Midpoint Rule):

$$I \approx h \cdot f\left(\frac{x_{j+1} + x_j}{2}\right)$$

Considering one segment of Simpson's integration given in the neighboring figure



$$S_i \approx \frac{h}{3}\left[f_i + 4f_{i+1} + f_{i+2}\right] \quad \text{--- (1)}$$

One can use Rectangle rule to write integral I

$$R_i \approx 2h \cdot f_{i+1} \quad \text{--- (2)}$$

Also, using trapezoidal Rule, same integral I (where $j = i$ & $j+1 = i+2$)

$$T_i \approx \frac{h}{2}\left[f_i + 2f_{i+1} + f_{i+2}\right] \quad \text{--- (3)}$$

Combining (1), (2) & (3) we can write.

$$S_i = \frac{2}{3}R_i + \frac{1}{3}T_i \quad \text{--- (4)}$$

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Q.1

However, looking at error estimations provided for Rectangular rule (eqn. 3.7 in Prof. Moin's book, pg 29) & similarly for trapezoidal rule (eqn. 3.8, pg 30) respective errors E_R & E_T can be expressed as :

$$E_R = \frac{h_i^3}{24} f''(x_{i+1}) + \frac{h_i^5}{1920} f^{iv}(x_{i+1}) + \cdots \quad — (5)$$

$$E_T = -\frac{1}{12} h_i^3 f''(x_{i+1}) - \frac{1}{480} h_i^5 f^{iv}(x_{i+1}) + \cdots \quad — (6)$$

if I is analytically correct integral we have,

$$I = S_i + E_S \ ; \quad I = T_i + E_T \ ; \quad I = R_i + E_R$$

i.e. $S_i = I - E_S$; $T_i = I - E_T$; $R_i = I - E_R$.

substitute above in (4)

$$E_S = \frac{2}{3} E_R + \frac{1}{3} E_T \quad —— (7)$$

Putting (5) & (6) in (7)

$$E_S = \frac{2}{3}\left[\frac{h_i^3}{24} f''_{i+1} + \frac{h_i^5}{1920} f^{iv}_{i+1} + \cdots\right] + \frac{1}{3}\left[-\frac{h_i^3}{12} f''_{i+1} - \frac{h^5}{480} f^{iv}_{i+1} + \cdots\right]$$

$$= \underbrace{\left[\frac{2}{3}\cdot\frac{1}{24} - \frac{1}{3}\cdot\frac{1}{12}\right]}_{=0} h^3 f''_{i+1} + \left[\frac{2}{3}\cdot\frac{1}{1920} - \frac{1}{3}\cdot\frac{1}{480}\right] h^5 f^{iv}_{i+1} + \cdots$$

$$= \frac{-1}{2880} h^5 f^{iv}_{i+1} + \cdots$$

$\boxed{\text{Local error} \propto h^5 \Rightarrow \text{Global error} \propto h^4 \text{ for Simpson's Method}}$

Q.2

Error Summary Table

		Local Error	Global. Err
A) Integral	Trapezoidal	$O(h^3)$	$O(h^2)$
	Rectangular	$O(h^3)$	$O(h^2)$
	Simpson's	$O(h^5)$	$O(h^4)$
	Gaussian Quadrature		
B) Differentiation	1st order Forward	$O(h)$	
	1st Order Backward	$O(h)$	
	1st order Central	$O(h^2)$	
	2nd order forward	$O(h^2)$	
	2nd Order backward	$O(h^2)$.	
	2nd Order Central.	$O(h^2)$	

Q.3

ⓐ Newton- Raphson evaluates both $f(x)$ & $f'(x)$. Hence entails more number of evaluation compared to Newton-Secant which needs only one (as it can carry forward previous node value)

ⓑ Newton-Raphson is suitable for higher dimensional problems

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Q.4

Let

$$y_1 = y$$

$$y_2 = y_1' \ (= y')$$

$$y_3 = y_2' \ (= y'')$$

$$\cancel{y_4' = y_3' = 3y'' - y = 3y_2 - y_1}$$

$$y_4 = y_3' = y'''$$

$$\cancel{y_1' = y_2}$$

$$\cancel{y_2' = y_3}$$

$$\Rightarrow \cancel{y_3' = 3y_2 - y_1}$$

$$\therefore y_3' = y_4$$

$$\cancel{y_3' = 3y_2 - y_1}$$

$$y_1' = y' = y_2$$

$$y_2' = y_1'' = y'' = y_3$$

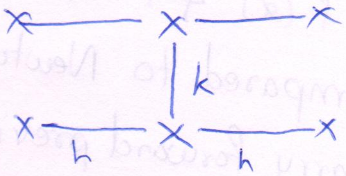
$$y_3' = y_2'' = y_1''' = y^{iv} = y_4$$

$$y_4' = 3y'' + y = 3y_3 + y$$

Q.5

(a) $\dfrac{k}{h^2} < \dfrac{1}{2}$ ∴ k is stepsize in time axis
h is stepsize in spatial (x-axis)

(b)



Crank-Nicolson scheme is better compared to Finite Diff. scheme as it has no restriction on $r = \frac{k}{h^2}$ like that of explicite method.

Numerical Methods -Assignment #2

Engineering Mathematics for Advanced Studies

IIT Dharwad

Autumn 2019

Submission - Thursday 15th Nov. 2019 5:30pm

Total score - 10 marks

Late penalty - 1 day late* 30%, 100% for more than a day (*starts from 5:31pm, 15th Nov. 2019!)

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1. Derive expression for error in Simpsons' integration method. (You may refer to the prescribed text where it could be readily available) (2 marks)

2. Fill in the table below given that the grid spacing value (increment) is h (2 marks)

Operation	Method	Order of Error
Integration	Trapezoidal	
	Rectangular	
	Simpsons	
	Gaussian Quadrature	
Differentiation	First order Forward Difference	
	First order Backward Difference	
	First order central Difference	
	Second order Forward Difference	
	Second order Backward Difference	
	Second order central Difference	

3. Compare Newton-Raphson Method to Secant method -
   (a) which one entails entails more number of evaluations of a function value for a given x_n (1 mark)
   (b) which one is suitable for use in higher dimensional problem (1 mark)

4. Express following differential equations as a system of first order differential equations (2 marks)

$$\frac{d^4y}{dx^4} - 3\frac{d^2y}{dx^2} + y = 0$$

5. Regarding Parabolic PDE and Hyperbolic PDE, please read the text from the textbook.
   (a) While applying finite difference method to parabolic equations, what is the criterion for stability of the scheme? (1 mark)
   (b) Draw stencil for the Crank-Nicolson scheme and state advantage of Crank-Nicolson scheme against finite difference approximation of the parabolic PDE equation? (1 mark)