Numerical Methods -Assignment #1

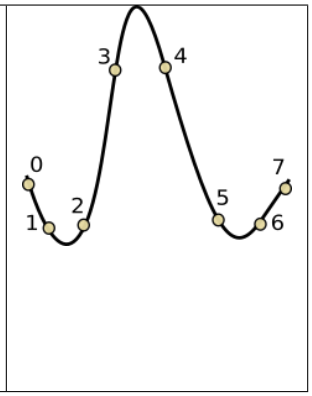
Engineering Mathematics for Advanced Studies
IIT Dharwad
Autumn 2019

Submission - Thursday 14th Nov. 2019 5:30pm
Total score - 10 marks
Late penalty - 1 day late* 30%, 100% for more than a day (*starts from 5:31pm, 14th Nov. 2019!)

NOTE - Please use some software tool to solve the system of linear equations. Minimum manual work is expected.

1. Given velocity data:

No.	Time	Velocity
0	0.0	0.12507
1	2.0	-0.12828
2	4.0	-0.10022
3	6.0	0.66500
4	8.0	0.66500
5	10.0	-0.10022
6	12.0	-0.12828
7	14.0	0.12507



Please define cubic spline for this data with free run out condition.
(Hint: Refer to equation 1.7 in Parviz Moin book)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{h}{6} & \frac{2h}{3} & \frac{h}{6} & 0 & 0 & 0 & 0 \\ & & & & & & 0 \\ & & & \ddots & & & \\ 0 & & & & & & \\ 0 & 0 & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} g"(x_0) \\ g"(x_1) \\ \\ \vdots \\ \\ g"(x_6) \\ g"(x_7) \end{Bmatrix} = \begin{Bmatrix} a \\ \frac{f(x_2)-f(x_1)}{h} - \frac{f(x_1)-f(x_0)}{h} \\ \\ \vdots \\ \\ \\ b \end{Bmatrix}$$

   (a) Leverage the fact that datapoints are equidistant in time axis i.e. $h_i = \Delta_i = constant =$? (marks 1)

   (b) What are the values of a and b in above set-up worked out for your reference? (marks 1)

   (c) Populate the matrix appropriately refereing to equation 1.7 in Prof. Parviz Moin's book and solve. (marks 2)

   (d) Use the $g"(x_i)$ information found above to get 7 piecewise functions $g_i(x) \quad for \quad i = 1 \; to \; 7$ given by equation 1.6 in the same book. (marks 2)

2. Considering points 0,1,2,3 in above data, please write Langrange polynomial. (marks 1)

3. Evalate the slope all the points using Pade scheme (system of linear eqns - eqn 2.18 in Moin book) : (marks 1)

4. **Given the tabulated data above**, if one is interested to get the distance traveled, i.e. area under the velocity-time curve, which of the following is more suitable method and why (think about pros and cons of each method):

   (a) Rectangular (not discussed in class)

   (b) Trapezoidal

   (c) Simpson's method

   (d) Gauss quadrature (marks 1)

5. Integrate above function using (b) above. (marks 1)

6. For those more interest (no marks) -

   (a) Plot above interpolation for 0 to 7 node (full range) Lagrange interp with Cubic spline interpolation and compare by superimposion of plots (Use of some software is expected to code and plot)

   (b) See the sensitivity of the cubic spline to the end conditions - We used Run out spline above. What if it was parabolic end condition? How matrix set-up would change in case of parabolic condition?

   (c) Compare above results with analytical results sinc function ($f(t) = \frac{sin(t_o)}{t_o}$) with offeset $t = t_o + 7.5$

   (d) For numerical integration - which one is expected to be more accurate Rectangular or Trapezoidal

   (e) What is the noticeable characteristic of the matrix used in the system of linear equations given above?