Abstract. Bounded model checking (BMC) is an efficient formal verification technique which allows for desired properties of a software system to be checked on bounded runs of an abstract model of the system. The properties are frequently described in some temporal logic and the system is modeled as a state transition system. In this paper we propose a novel counting logic, $L_C$, to describe the temporal properties of client-server systems with an unbounded number of clients. We also propose two dimensional bounded model checking ($2D$-BMC) strategy that uses two distinguishable parameters, one for execution steps and another for the number of tokens in the net representing a client-server system, and these two evolve separately, which is different from the standard BMC techniques in the Petri Nets formalism. This $2D$-BMC strategy is implemented in a tool called DCModelChecker which leverages the $2D$-BMC technique with a state-of-the-art satisfiability modulo theories (SMT) solver Z3. The system is given as a Petri Net and properties specified using $L_C$ are encoded into formulas that are checked by the solver. Our tool can also work on industrial benchmarks from the Model Checking Contest (MCC). We report on these experiments to illustrate the applicability of the $2D$-BMC strategy.

Keywords: Bounded Model Checking · SAT solvers · Petri Nets · Counting Logics · Temporal Logics

1 Introduction

Model checking \cite{baier2008principles} is a formal verification technique that allows for desired behavioral properties of a given system to be verified based on a suitable model of the system through systematic inspection of all states of the model. The major challenge of model checking is that the state space of systems might be infinite. In bounded model checking, this challenge is overcome by assuming a predetermined bound on the runs, and all possible paths are explored. This technique uses a single parameter for execution steps, to unfold the system behaviour. Bounded Model Checking \cite{finkbeiner2004bounded,sebastiani2004unbounded} is widely used in the industry by restricting...
the model checking problem to a bounded problem and verifying properties on bounded runs of the system.

We consider the case of a client-server system with an unbounded number of agents, which we model as unbounded Petri Nets. In this work, we propose a novel extension to standard bounded model checking technique, the two-dimensional bounded model checking (2D-BMC) strategy for unbounded client-server systems, that uses two distinguishable parameters, one for execution steps and another for the number of tokens in the net and these two evolve separately, which is different from the standard BMC techniques for Petri Nets.

Typically, in standard BMC, temporal logic is used to specify the properties of the system. In this setting of unbounded client-server systems, we would also like to express additional counting properties of the client and the server. We propose a novel counting logic with temporal operators, $L_C$.

To illustrate the usefulness of the newly introduced 2D-BMC strategy and counting logic $L_C$, we implemented them in our new tool DCModelChecker. BMC for Petri Nets while expressing properties using linear temporal logic while leveraging the power of SMT Solvers remains unexplored. DCModelChecker attempts to bridge this gap. We have considered benchmark models from the Model Checking Contest [17], and translated a subset of the properties into $L_C$ and have obtained the satisfaction or counterexample trace in each case. Details of the tool and experiments are discussed in Section 5.1 and Section 6 respectively.

This paper makes the following contributions:

- We introduce a novel technique - 2D-BMC to verify the properties of the system.
- We introduce a counting logic language, $L_C$ for describing the counting properties of the system as well as temporal properties of the system.
- We build a tool, DCModelChecker, that uses 2D-BMC and the designed counting logic language for verification.
- We perform an experimental evaluation of our tool, with academic and industrial models from the Model Checking Contest (MCC) [17], and compare the results against the state of the art tool ITS-Tools [26]. To the best of our knowledge, ours is the only tool that can perform 2D-BMC and provide the counterexample as well.

**Organization of our paper:** We begin with the preliminaries on Petri Nets and their encoding in the next section. We introduce counting logic $L_C$ in Section 3. In Section 4, we discuss the novel 2D-BMC strategy and give its encoding and demonstrate the unfolding of formulas in this technique. The rest of the paper discusses the details of the tool DCModelChecker, elaborating on its architecture in Section 5.1 and the workflow of the tool in Section 5.2. Experiments using DCModelChecker are in Section 6 followed by the related work in Section 7 and future work in Section 8.

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3 https://doi.org/10.6084/m9.figshare.19611477.v3
2 Preliminaries

The bounded model checking problem can be described as follows. Given a system $S$ and a specification or property $\alpha$ on bounded runs of the system, decide whether system $S$ satisfies specification $\alpha$. This consists of checking that all runs of $S$ constitute models for $\alpha$. It suffices to show that no run of $S$ is a model for $\neg \alpha$, which is the same as checking that the intersection of the language accepted by $S$ and the language defined by $\neg \alpha$ is empty.

We illustrate how client-server systems with unbounded agents can be mathematically encoded as nets in Section 2.1 and in Section 3 we explore a suitable logic to express its properties.

It is to be noted that theoretically, there may be an unbounded number of clients in the unbounded client-server system at any instant. Practically, factors such as environment limitations and hardware design may limit the number of clients that are present in an instant.

2.1 Petri Nets

A Petri Net structure is a tuple $N = (P, T, F, W)$ where $P$ is a finite set of places, $T$ is a finite set of transitions, $F \subseteq (P \times T) \cup (T \times P)$ is the flow relation and $W : F \rightarrow \mathbb{N}_0$ is a weight function, where $\mathbb{N}_0$ is the set of non-negative integers. For any place (resp. transition) $z$ of the set $P \cup T$, the set $\{x \mid (x, z) \in F\}$ is called pre-transitions (resp. pre-places) of $z$, and the set $\{x \mid (z, x) \in F\}$ is called post-transitions (resp. post-places) of $z$.

A marking $M$ of a Petri Net is a function $M : P \rightarrow \mathbb{N}_0$. A Petri Net (system) is a tuple $PN = (N, M_0)$ where $N$ is a Petri Net structure and $M_0$ is an initial marking. A transition $t$ is enabled at marking $M$, if for each pre-place $p$ of $t$ we have $M(p) \geq W(p, t)$. A new marking is obtained when an enabled transition is fired, and is obtained by removing $W(p, t)$ tokens from each pre-place $p$ of $t$, and adding $W(t, p)$ tokens to each post-place $p$ of $t$, leaving tokens in the remaining places as it is.

Encoding of Petri Nets For the purpose of BMC we need to first encode a Petri Net system in propositional logic. A similar encoding can be found in [12]. Let $N = (P, T, F, W)$ be a Petri Net structure. Let $T = \{t_1, \ldots, t_n\}$ and $P = \{p_0, \ldots, p_l\}$. The variables used in the encoding are as follows. For each transition $t_i \in T$, there is a copy of the variable that is used in the encoding. For each place $p_i \in P$, we have two variables $p_{xi}$ and $p_{yi}$ in the encoding – $p_{xi}$ denotes the number of tokens in $p_i$ before firing of some transition and $p_{yi}$ denotes the number of tokens in the same place after the firing. We have a vector of variables $M$ that keep track of the marking. The propositional encoding of $N$ is defined by the formula $T = T_{\text{enabled}} \land T_{\text{firability}} \land T_{\text{next}}$, where:

- the formula $T_{\text{enabled}}$ states that more than one transition can be enabled at a time. i.e., it is an or over the preconditions of all enabled transitions and is given by $T_{\text{enabled}} = \pre_{t_0} \lor \pre_{t_1} \lor \cdots \lor \pre_{t_n}$;
the formula $T_{\text{next}}$ gives us the next transition that will be fired, and is expressed as an or expression over each transition and its postcondition and all other transitions and postconditions are negated.

$$T_{\text{next}} = (t_0 \land \neg t_1 \land \cdots \land \neg t_n \land \neg \text{post}_t_0 \land \cdots \land \neg \text{post}_t_n) \lor$$

$$(-t_0 \land t_1 \land \cdots \land \neg t_n \land \text{post}_t_0 \land \cdots \land \neg \text{post}_t_n) \lor \cdots \lor$$

$$(-t_0 \land \neg t_1 \land \cdots \land \neg t_n \land \text{post}_t_0 \land \cdots \land \text{post}_t_n);$$

the formula $T_{\text{firability}}$ relates the precondition of a transition with its distinct postcondition and is given as

$$T_{\text{firability}} = (\text{post}_t_0 \rightarrow \text{pre}_t_0) \land (\text{post}_t_1 \rightarrow \text{pre}_t_1) \land \cdots \land (\text{post}_t_n \rightarrow \text{pre}_t_n);$$

where for each $t_i$, $\text{pre}_t_i$ is a propositional formula defined over place variables encoding that the precondition of firing $t_i$ is satisfied and $\text{post}_t_i$ is a formula defined over place variables encoding the update in tokens after firing $t_i$ is satisfied.

We denote the encoding of Petri Net system $PN = (N, M_0)$ by $M = (T, M_0)$ where $M_0$ is the initial assignment of the marking vector $M$.

3 Counting Logic

Our next objective is to identify a suitable logical language for describing properties of the unbounded client-server system. Linear Temporal Logic (LTL) \[22,28\] is a natural choice to describe temporal properties of systems.

While LTL is useful to us, we cannot express any counting properties, hence, we propose the counting logic language to specify and verify the unbounded client-server system. Logic $L_C$ is an extension of LTL with a few differences. In the case of LTL, atomic formulas are propositional constants which have no further structure. A monodic formula is a well formed formula with at most one free variable in the scope of a temporal modality. In $L_C$, there are three types of atomic formulas: (1) describing basic server properties, $P_s$ which are propositional constants (2) counting sentences of the kinds $(\#x > c) \alpha$ and $(\#x \leq c) \alpha$ over client properties and $c$ is a non-negative integer, denoting the number of clients in $\alpha$, and (3) comparing sentences of the kinds $(\#x) \alpha \leq \beta$ and $(\#x) \alpha > \beta$.

Formally, the set of client formulas $\Delta$ is given by:

$$\alpha, \beta \in \Delta ::= (\#x > c)p(x) \mid (\#x \leq c)p(x) \mid (\#x)p(x) \leq q(x) \mid (\#x)p(x) > q(x) \mid \alpha \lor \beta \mid \alpha \land \beta$$

where $p, q \in P_c$ and $c$ is a non-negative integer, as noted above. While $\Delta$ does not contain an explicit $= \alpha$ operator, it is natural to express equality of $p(x)$ and $q(x)$ as follows: $(\#x)p(x) > q(x) \land (\#x)q(x) > p(x)$.

The server formulas are defined as follows:

$$\psi \in \Psi ::= q \in P_s \mid \varphi \in \Delta \mid \neg \psi \mid \psi \lor \psi \mid \psi \land \psi \mid X\psi \mid F\psi \mid G\psi \mid \psi_1 U \psi_2$$

Modalities $X, F, G$ and $U$ are the usual modal operators: Next, Eventually, Globally and Until respectively.
**Semantics** The logic is interpreted over model sequences. Formally, a model is a sequence $\rho = m_0, m_1, \ldots$, where for all $i \geq 0$ we have a triple $m_i = (\nu_i, V_i, \xi_i)$ such that:

1. $\nu_i \subseteq P_i$, gives the local properties of the server at instant $i$.
2. $V_i \subseteq C N$ gives the clients alive at instant $i$, where $C N$ is a countable set of client names that can be assigned to the processes in the system. Further, for all $i \geq 0$, $V_{i+1} \subseteq V_i$ or $V_i \subseteq V_{i+1}$.
3. $\xi_i : V_i \rightarrow 2^{F^c}$ gives the properties satisfied by each live agent at the $i$th instant.

The truth of a formula at an instant in the model is given by the relations $\models$ and $\models_{\Delta}$ defined by induction over the structure of $\psi$ and $\alpha$ respectively as follows:

1. $\models_{\nu_i} \varphi$, Note that $q$’s denote atomic local server propositions.
2. $\models_{\nu_i} \varphi$ if $\varphi$ is a sentence from the set of client formulae $\Delta$. In order to define the satisfiability of $\varphi$, we need to use the rules defined for the relation $\models_{\Delta}$.
3. $\models_{\nu_i} \neg \psi$ iff $\models_{\nu_i} \varphi$.
4. $\models_{\nu_i} \varphi \lor \varphi'$ iff $\models_{\nu_i} \varphi$ or $\models_{\nu_i} \varphi'$.
5. $\models_{\nu_i} \varphi \land \varphi'$ iff $\models_{\nu_i} \varphi$ and $\models_{\nu_i} \varphi'$.
6. $\models_{\nu_i} X \psi$ iff $\models_{\nu_{i+1}} \psi$.
7. $\models_{\nu_i} G \psi$ iff $\forall j \geq i, \models_{\nu_j} \psi$.
8. $\models_{\nu_i} \psi_1 U \psi_2$ iff $\exists j \geq i, \models_{\nu_j} \psi_1$ and for all $i \leq j' < j : \models_{\nu_j'} \psi_2$.
9. $\models_{\nu_i} (#x > c)p(x)$ holds in model $\nu_i$ at instance $i$ if there are strictly more than $\xi_i$ clients that satisfy the property $\nu_i$ at instance $i$.
10. $\models_{\nu_i} (#x \leq c)p(x)$ holds in model $\nu_i$ at instance $i$ if there are less than $\xi_i$ clients that satisfy the property $\nu_i$ at instance $i$.

To illustrate the expressibility of the counting logic, consider the property of an infinite state system, such that at some time in the future, the number of tokens at place $p_1$ is greater than the number of tokens in place $p_0$, is written as $F(\#x)p(x) > p_0(x)$. Details of expressing the properties with respect to case studies are discussed in Appendix A.2 and Appendix B.2.
4 The Two Dimensional Bounded Model Checking
Strategy

In this section, we set out to enunciate the novel two dimensional bounded model checking strategy that we have devised and implemented to check $\mathcal{L}_\mathcal{C}$ specifications against Petri Net models. To quote Biere et al. [6], “The basic idea behind bounded model checking (BMC) is to restrict the general model checking problem to a bounded problem. Instead of asking whether the system $M$ violates the property $\Psi$, we ask whether the system $M$ has any counterexample of length $k$ to $\Psi$. This bounded problem is encoded into SAT.” In 2D-BMC, we ask whether the system $M$ has any counterexample of length $\lambda$ and number of clients $\kappa$ to $\Psi$, in the unbounded client-server setting, and encode this bounded problem into SMT.

First, we describe the notation that we have used. Let $\mathcal{M}$ be the propositional encoding of the Petri Net model of the system. Also, let $\phi$ be the $\mathcal{L}_\mathcal{C}$ property that we want to verify. As usual we negate the property $\phi$ and let $\psi = \neg \phi$. We also assume that $\psi$ is used in its negation normal form.

The 2D-BMC encoding of $\mathcal{M}$ against $\psi$ for the bound $k = \lambda + \kappa$ (where $k \geq 0$) is denoted by $[\mathcal{M}, \psi]_{(\lambda, \kappa)}$ and defined as follows:

$$[\mathcal{M}, \psi]_{(\lambda, \kappa)} = [\mathcal{M}]_{(\lambda, \kappa)} \land \left( (\neg L_{(\lambda, \kappa)} \land [\psi]^0_{(\lambda, \kappa)}) \lor \bigvee_{i=0}^{k} (i L_{(\lambda, \kappa)} \land i [\psi]^0_{(\lambda, \kappa)}) \right)$$

The bound $k$ has two parts $\lambda$ and $\kappa$: $\lambda$ gives the bound for time instances and $\kappa$ gives the bound for number of clients. $[\mathcal{M}]_{(\lambda, \kappa)}$ is the propositional formula encoding the runs of $\mathcal{M}$ of bound $k = \lambda + \kappa$. The formulae $[\mathcal{M}]_{(\lambda, \kappa)}$, for $0 \leq \lambda, \kappa \leq k$ are defined later in this section.

The formulas $i L_{(\lambda, \kappa)} (0 \leq i \leq \lambda)$ and $L_{(\lambda, \kappa)}$ are loop conditions that are mentioned in the encoding above. For any $0 \leq i \leq \lambda$, $i L_{(\lambda, \kappa)} = T(s_i, s_i)$ where $s_i$ is the $i$th state in the run of $\mathcal{M}$ and $s_\lambda$ is the $\lambda$th state. Note, here state is defined in terms of value of the marking vector $M$. So, for any instance $i$, $s_i$ corresponds to the state of vector $M$ at $i$. Clearly, when $i L_{(\lambda, \kappa)}$ holds it means there is a transition from $s_i$ to $s_i$ which denotes a back loop to the $i$th state. The other loop condition is defined as follows: $L_{(\lambda, \kappa)} = \lor_{0 \leq i \leq \lambda} i L_{(\lambda, \kappa)}$. When $L_{(\lambda, \kappa)}$ holds, it means there is a back loop to some state in the bounded run of $\mathcal{M}$.

Formula $[\psi]^0_{(\lambda, \kappa)}$ is the propositional encoding of $\psi$ for the bound $k = \lambda + \kappa$ when $\psi$ is asserted at initial instance $i = 0$ and there is no loop in the run of $\mathcal{M}$. On the other hand, $i [\psi]^0_{(\lambda, \kappa)}$ is the propositional encoding of $\psi$ for the bound $k = \lambda + \kappa$ when $\psi$ is asserted at initial instance $i = 0$ and there is a back loop to the $i$th state in the run of $\mathcal{M}$. Formulas $[\psi]^i_{(\lambda, \kappa)}$ and $i [\psi]^i_{(\lambda, \kappa)}$, for any $i$, are extensions of similar mappings defined in [5].

The bound $k = \lambda + \kappa$ starts from 0 and is incremented by 1 in each (macro-)step. For a fixed $k$, $\lambda$ may start from 0, incremented by 1 in each (micro-)step till $k$. Simultaneously, $\kappa$ may move from $k$ to 0 and decrement by 1 in each
We look at each (micro-)step. Let the variables being used in the Boolean encoding of Petri Net be:

- Transitions: \(t_0, t_1, \ldots, t_n\), and Places: \(p_0, p_1, \ldots, p_n\).

We use copies of transition variables \(t_0, \ldots, t_n\), place variables \(p_0, \ldots, p_n\), where \(0 \leq i \leq \lambda\), in \([M]_{(\lambda, \kappa)}\). For any \(\kappa \geq 0\), we define

\[
[M]_{(0, \kappa)} = I(s_0) \land (\bigwedge_{0 \leq j \leq n_p} p_j \leq \kappa).
\]

Inductively, for any \(\lambda > 0\),

\[
[M]_{(\lambda, \kappa)} = [M]_{(\lambda-1, \kappa)} \land (T(s_{\lambda-1}, s_\lambda) \land (\bigwedge_{0 \leq j \leq n_p} p_j \leq \kappa)).
\]

Now that we have formally described the notations for encoding of \(M\), we give the propositional encoding of \(L_C\) for \(M\) in the subsequent section.

### 4.1 Propositional encoding of the logic \(L_C\)

We need to introduce counter variables for each place \(p\) in the input Petri Net in order to define the propositional encodings of the property \(\psi\). These extra variables are as follows: \(\{c^p_i | i \geq 0, p \text{ is a place in the Petri Net}\}\). Now we are ready to define \(l[\psi]_{(\lambda, \kappa)}\) and \(l[\psi]_{(\lambda, \kappa)}\). We define \(l[\psi]_{(\lambda, \kappa)}\) inductively as follows:

1. \(l[q]_{(\lambda, \kappa)} = q_i\)
2. \(l[q]_{(\lambda, \kappa)} = \neg q_i\)
3. \(l[q_1 \lor q_2]_{(\lambda, \kappa)} = l[q_1]_{(\lambda, \kappa)} \lor l[q_2]_{(\lambda, \kappa)}\)
4. \(l[q_1 \land q_2]_{(\lambda, \kappa)} = l[q_1]_{(\lambda, \kappa)} \land l[q_2]_{(\lambda, \kappa)}\)
5. \(l[X\psi]_{(\lambda, \kappa)} = \bigwedge_{j=\min(I,i)} l[\psi]_{(\lambda, \kappa)}\)
6. \(l[F\psi]_{(\lambda, \kappa)} = \bigwedge_{j=\min(I,i)} l[\psi]_{(\lambda, \kappa)}\)
7. \(l[G\psi]_{(\lambda, \kappa)} = \bigwedge_{j=\min(I,i)} l[\psi]_{(\lambda, \kappa)}\)
8. \(l[U\psi]_{(\lambda, \kappa)} = \bigwedge_{j=\min(I,i)} l[\psi]_{(\lambda, \kappa)}\)
9. \(l[(\#x < c)p(x)]_{(\lambda, \kappa)} = c^p_i > c\)
10. \(l[(\#x \leq c)p(x)]_{(\lambda, \kappa)} = c^p_i \leq c\)
11. \(l[(\#x)p(x) > q(x)]_{(\lambda, \kappa)} = c_i^p > c_q\)
12. \(l[(\#x)p(x) \leq q(x)]_{(\lambda, \kappa)} = c_i^p \leq c_q\)
13. \(l[\alpha_1 \lor \alpha_2]_{(\lambda, \kappa)} = l[\alpha_1]_{(\lambda, \kappa)} \lor l[\alpha_2]_{(\lambda, \kappa)}\)
14. \( i(\alpha_1 \lor \alpha_2)_{(\lambda, \kappa)}^i \equiv i(\alpha_1)_{(\lambda, \kappa)}^i \lor i(\alpha_2)_{(\lambda, \kappa)}^i \)

We define \([\psi]_{(\lambda, \kappa)}^i\) inductively as follows:

1. \([q]_{(\lambda, \kappa)}^i \equiv q_i\)
2. \([-q]_{(\lambda, \kappa)}^i \equiv \lnot q_i\)
3. \([\psi_1 \lor \psi_2]_{(\lambda, \kappa)}^i \equiv [\psi_1]_{(\lambda, \kappa)}^i \lor [\psi_2]_{(\lambda, \kappa)}^i\]
4. \([\psi_1 \land \psi_2]_{(\lambda, \kappa)}^i \equiv [\psi_1]_{(\lambda, \kappa)}^i \land [\psi_2]_{(\lambda, \kappa)}^i\]
5. \([X\psi_1]_{(\lambda, \kappa)}^i \equiv \left\{ \begin{array}{ll} [\psi_1]_{(\lambda, \kappa)}^{i+1} & \text{if } i < \kappa \\ [\psi_1]_{(\lambda, \kappa)}^i & \text{if } i = \kappa \end{array} \right.\]
6. \([F\psi_1]_{(\lambda, \kappa)}^i \equiv \bigvee_{j=\min(l,i)}^{\kappa} [\psi_1]_{(\lambda, \kappa)}^j\]
7. \([G\psi_1]_{(\lambda, \kappa)}^i \equiv \bigwedge_{j=\min(l,i)}^{\kappa} [\psi_1]_{(\lambda, \kappa)}^j\]
8. \([\psi_1 U\psi_2]_{(\lambda, \kappa)}^i \equiv \bigvee_{j=i}^{\kappa} ([\psi_2]_{(\lambda, \kappa)}^j \land \bigwedge_{n=i}^{j-1} [\psi_1]_{(\lambda, \kappa)}^n)\]
9. \([(\#x > c)p(x)]_{(\lambda, \kappa)}^i \equiv c_p^i > c\]
10. \([(\#x \leq c)p(x)]_{(\lambda, \kappa)}^i \equiv c_p^i \leq c\]
11. \([(\#x)p(x) > q(x)]_{(\lambda, \kappa)}^i \equiv c_p^i > c_q^i\]
12. \([(\#x)p(x) \leq q(x)]_{(\lambda, \kappa)}^i \equiv c_p^i \leq c_q^i\]
13. \([\alpha_1 \lor \alpha_2]_{(\lambda, \kappa)}^i \equiv [\alpha_1]_{(\lambda, \kappa)}^i \lor [\alpha_2]_{(\lambda, \kappa)}^i\]
14. \([\alpha_1 \land \alpha_2]_{(\lambda, \kappa)}^i \equiv [\alpha_1]_{(\lambda, \kappa)}^i \land [\alpha_2]_{(\lambda, \kappa)}^i\]

To determine the encoding of \([M]_{(0, \kappa)}\), we need to perform \textbf{and} operation on the initial state and the place constraints. The place constraints are due to the \(\kappa\) variable, for each of the places.

\[ [M]_{(0, \kappa)} = I(s_0) \land \bigwedge_{0 \leq j \leq n_p} p_{j0} \leq \kappa. \]

Similarly, to determine \([M]_{(\lambda, \kappa)}\), we can make use of the previously encoded \([M]_{(\lambda-1, \kappa)}\) and unfold the transition relation by one length, and \textbf{and} it with the place constraints to obtain the following relation:

\[ [M]_{(\lambda, \kappa)} = [M]_{(\lambda-1, \kappa)} \land (T(s_{\lambda-1}, s_{\lambda}) \land \bigwedge_{0 \leq j \leq n_p} p_{j\lambda} \leq \kappa). \]

The encoding of the \(\lnot, \land, \lor\) operators are straightforward. We also define the encoding using the temporal operators. Notice that \([(\#x > k)p(x)]_{(\lambda, \kappa)}^i\) is encoded using the counter \(c_p\) at the place \(p\), and this is translated as \(c_p > \kappa\). Similarly, we have \([(\#x > k)p(x)]_{(\lambda, \kappa)}^i \equiv c_p > \kappa.\]
Unfolding the encoded formula  The unfolding of the formula \((M, \psi)_{(\lambda, \kappa)}\) for each \(k\), upto \(k = 2\) is depicted pictorially in Fig. 1. Initially, when \(k = \lambda + \kappa = 0\), \(k = 0\) (bound), \(\lambda = 0\) (time instance), \(\kappa = 0\) (number of clients), the formula \((M, \psi)_{(0,0)}\) is evaluated. If this is found to be satisfiable, then, a witness is obtained, and we are done. If not, in the next macro-step of the unfolding, where \(k = \lambda + \kappa = 1\), there are possibly two micro-steps to be explored \((\lambda, \kappa) = (0, 1)\) and \((\lambda, \kappa) = (1, 0)\). Firstly, \(\kappa\) is incremented and the formula \((M, \psi)_{(0,1)}\) is evaluated. If this found to be unsatisfiable, \(\lambda\) is incremented and the formula \((M, \psi)_{(1,0)}\) is evaluated. In each micro-step of the unfolding, either \(\lambda\) or \(\kappa\) are incremented, and the resulting formula is verified. Details of the unfolding with respect to the tool are in Section 5.2.

5 The 2D-BMC Model Checker Tool: DCModelChecker

DCModelchecker is an easy-to-use tool for performing two dimensional bounded model checking of Petri Nets using the logic \(L_C\). We give the architecture and the overview of the tool in Section 5.1. We describe the workflow in Section 5.2 and in Section 6, we report the experiments. The experiments are easily reproducible by directly executing the scripts in our artifact. It is a self-contained virtual machine image — additional installations or tools are unnecessary for replicating our results.
5.1 Architecture

We introduce a tool to perform two dimensional bounded model checking, DC-ModelChecker. While several tools perform bounded model checking, ours is unique in the model checking strategy as well as displaying the counterexample. We describe the details of related work in the Section 7.

![DCModelChecker architecture](image)

The general architecture of the tool is shown in Fig. 2. Our tool has two primary inputs—system description in standard PNML format and the property to be tested expressed in counting logic $\mathcal{L}_C$. The system description using Petri Nets in PNML format can be obtained from the vast collection of industrial and academic benchmarks available at MCC [17] or created using a Petri Net Editor [30]. We make use of MCC benchmarks, for comparative testing. Currently, we have translated a subset of the MCC properties into $\mathcal{L}_C$.

Firstly, the two inputs are fed to the pre-processing module and consequently to the DCModelChecker tool. The objective of the pre-processing Module is to read and validate the two inputs. We make use of ANTLR [20] to achieve this. We give the grammar of the nets and properties to the tool so that it can recognize it against its respective grammar as shown in Fig. 3. ANTLR generates a parser for that language that can automatically build parse trees representing how a grammar matches the input. The parse trees can be walked to construct the data structures that our tool requires. We have manually written the grammars for the PNML format of both types and the grammar of $\mathcal{L}_C$, to be used by ANTLR.

Our tool reads the output of the pre-processing module and checks if the model satisfies the property or not. We make use of the Z3 SAT/SMT Solver [19], to solve the encoded formula and give us unsat, or sat with a counterexample trace. If unsat, the tool can increment the bound and look further, until the external termination bound is hit. We picked Z3, for its wide industrial applications, developer community support, and ease of use. The detailed workflow is discussed in the subsequent section.

5.2 Workflow

The system description $M$, the property formula $\phi$, and external termination bound $k$ are given to us. First, we negate the property, $\psi = \neg \phi$. Note that the negation normal form of $\psi$ is always used in the BMC process. For the bound
$k = 0$ and the corresponding micro-step $\langle \lambda, \kappa \rangle = \langle 0, 0 \rangle$, we construct the formula $[M, \psi]_{\langle 0, 0 \rangle}$ and feed it to the solver.

If the above formula is satisfiable, the property $\phi$ is violated in the initial state and we have a witness at $k = 0$ and we can stop our search. If the base case is unsatisfiable, we consider the next micro-step $\langle \lambda, \kappa \rangle$ – with the bound $k = \lambda + \kappa = 1$ – and construct the formula $[M, \psi]_{\langle \lambda, \kappa \rangle}$ and feed it to the solver. If this formula is satisfiable, the property $\phi$ is violated for $k = \lambda + \kappa$ and we have a witness for this $k$ and we can stop our search. Otherwise, we may continue with the next micro-step $\langle \lambda', \kappa' \rangle$ and so on. The order in which the micro-steps are considered is illustrated in Fig. 1. We shall continue until we have considered all micro-steps for the termination bound $k$.

6 Experiments

To illustrate the expressibility of $\mathcal{L}_C$ and the applicability of 2D-BMC, we performed two phases of experiments, their results are described in this section. Firstly, we considered benchmarks models from the Model Checking Contest [17] and expressed the properties in $\mathcal{L}_C$. We considered the following property categories and translated them to $\mathcal{L}_C$ — LTLFireability, LTLCardinality, ReachabilityFireability, ReachabilityCardinality. In particular, the results of verifying a subset of properties are consolidated in Table 1. The table shows the number of sat and unsat properties in each property category (consisting of 16 properties each per category), and the time taken to execute (in ms) for DCModelChecker.
as well as the state-of-the-art model checking tool, ITS-Tools [26]. Our tool DCModelChecker was executed with the bound 1. ITS-Tools was executed with a timeout of 600 seconds. It can be observed that the DCModelChecker performs well in some property categories. While some properties were unsat for the above bound and timeout period constraints, they may have become sat, on incrementing the bound. Hence there is a difference in the number of sat and unsat properties found by DCModelChecker versus ITS-Tools. For LTL Fireability and LTL Cardinality properties, DCModelChecker takes comparatively less execution time than ITS-Tools. However, the number of sat and unsat properties differ slightly. As part of future work, we would like to identify the property types where DCModelChecker has an advantage. Our tool DCModelChecker is available online [21] and the results in this section can be easily reproduced.

<table>
<thead>
<tr>
<th>Model Name</th>
<th>Property Category</th>
<th>DCModelChecker</th>
<th>ITS-Tools</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>sat</td>
<td>unsat</td>
<td>time(s)</td>
</tr>
<tr>
<td>Dekker-PT-010</td>
<td>LTL Cardinality</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>LTL Fireability</td>
<td>2</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Reachability Cardinality</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Reachability Fireability</td>
<td>0</td>
<td>16</td>
</tr>
</tbody>
</table>

Table 1. Results of comparative testing

Secondly, we considered two infinite-state systems — firstly, the Autonomous Parking System (APS) and secondly, the Unbounded Process Scheduler (UPS), modeled as a net, primarily to take a closer look at how exactly the 2D-BMC strategy works, given that UPS is an extensively researched system [18] and has a much smaller net representation. The APS and UPS case studies are discussed in Appendix A and Appendix B respectively. Some of the properties of the Autonomous Parking System verified using DCModelChecker are given in Table 2. A sample of the properties that were written using the counting logic $L_C$ for the UPS to demonstrate its expressibility are in Table 3. It is natural to see that we can expand this to more complex properties.

With respect to the UPS, consider the property $F (\# x p_1(x) > p_0(x))$, written in $L_C$. It is a counting property with a finally operator at the root. Hence, this property gets violated when there is an instance where the number of processes at place $p_1$ is less than the number of processes/tokens at place $p_0$. In the model, this happens at time instant $\lambda = 3$, when the number of processes $\kappa = 1$, and a counterexample trace is obtained as well.

To the best of our knowledge, ours is the only tool that uses 2D-BMC in the Petri Net setting, for an extension of linear temporal logic while making use of an SMT solver and gives a counterexample trace. In Section 7 we discuss the contributions of other tools that verify the properties of Petri Nets.
Table 2. Results of testing APS using DCModelChecker

<table>
<thead>
<tr>
<th>S.No</th>
<th>UPS Property</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$F((#x)p_1(x) &gt; p_0(x))$</td>
<td>$k = 4, \kappa = 1, \lambda = 3$, SAT.</td>
</tr>
<tr>
<td>2</td>
<td>$G((#x)p_2(x) &lt;= p_1(x) &lt;= p_0(x))$</td>
<td>$k = 3, \kappa = 1, \lambda = 2$, SAT.</td>
</tr>
<tr>
<td>3</td>
<td>$G((#x_0\land t_1 \land t_7)$</td>
<td>$k = 0, \kappa = 0, \lambda = 0$, SAT</td>
</tr>
<tr>
<td>4</td>
<td>$F((#x &lt; 1)p_2(x) \land (#x &lt; 3)p_1(x)$</td>
<td>External Termination Bound reached, UNSAT.</td>
</tr>
<tr>
<td>5</td>
<td>$F((#x &lt; 2)p_0(x)$</td>
<td>External Termination Bound reached, UNSAT.</td>
</tr>
</tbody>
</table>

Table 3. Results of testing UPS using DCModelChecker

<table>
<thead>
<tr>
<th>S.No</th>
<th>UPS Property</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$G(#x)p_2(x) &gt; p_3(x)$</td>
<td>$k = 1, \kappa = 1, \lambda = 0$, SAT.</td>
</tr>
<tr>
<td>2</td>
<td>$F((#x)p_2(x) &lt;= p_3(x) \land (#x)p_3(x) &lt;= p_2(x))$</td>
<td>External Termination Bound reached, UNSAT.</td>
</tr>
<tr>
<td>3</td>
<td>$G((#x &gt; 0)p_1(x)(#x &gt;= 0)p_7(x)(#x &gt; 0)p_8(x)$</td>
<td>External Termination Bound reached, UNSAT.</td>
</tr>
<tr>
<td>4</td>
<td>$F((#x)p_6(x) &gt; p_2(x))$</td>
<td>$k = 4, \kappa = 1, \lambda = 3$, SAT.</td>
</tr>
<tr>
<td>5</td>
<td>$((#x &gt; 0)p_1(x)(#x &gt; 0)p_7(x)(#x &gt; 0)p_8(x)(#x &gt; 0)p_7(x)\land (#x)p_3(x) &lt;= p_2(x)$</td>
<td>External Termination Bound reached, UNSAT.</td>
</tr>
</tbody>
</table>

7 Related Work


BMC using SMT Solvers has been an active area of research [24]. In particular, while applying BMC in the Petri Net setting, techniques such as net reductions, and structural reductions [27] are applied. Recently, in [12] the generalized reachability of Petri Net is encoded into a BMC problem and solved using SMT solvers.

However, BMC for Petri Nets while expressing properties using LTL and its extensions while leveraging the power of SMT Solvers remains unexplored. DC-ModelChecker attempts to bridge this gap while preserving the original Petri Net structure. In Section 4.1 we provide the propositional encoding of the counting logic $L_C$ which distinguishes our work. In [26] a similar counting linear temporal logic for specifying properties of multi-robot applications is proposed. In contrast, $L_C$ uniquely supports specifying properties of client-server systems.

The Model Checking Contest (MCC) [17] has attracted many tools for formal verification of concurrent systems, and also industrial and academic benchmarks. Tools participating in MCC, use portfolio approach and give answers in binary, based on the requirement of the contest. In particular, we looked at
ITS-Tools \cite{26} which has 99.99\% tool confidence across categories in the contest and makes use of structural reduction techniques \cite{27} to arrive at the solution quickly. ITS-Tools uses a layer of SMT when solving LTL, however, it does not perform BMC.

While ITS-Tools is much faster than our tool, it gives only binary answers (sat or unsat), as this conforms with the expectations of the MCC unlike our tool which gives a counterexample trace, if the property is not satisfied. The counterexample trace is quite useful for property verification. To the best of our knowledge, DCModelChecker is the only tool that performs bounded model checking for Petri Nets using an extension of linear temporal logic using SMT solvers and provides a counterexample trace, that can be easily interpreted. Currently, the goal of this tool is to demonstrate the usefulness of the 2D-BMC strategy and the logic $\mathcal{L}_C$. To ensure the correctness of the results, we executed the same benchmarks on ITS-Tools as well. The results of comparative testing are in Section 6. These experiments can be replicated using the scripts available in our artifact \cite{21}.

8 Conclusion and Future Work

We have successfully explored the reachability, fireability, and cardinality properties of the unbounded client-server systems using the proposed counting logic $\mathcal{L}_C$. We have also proposed a novel bounded model checking strategy 2D-BMC applicable for Petri Nets. We have implemented a tool DCModelChecker that uses the proposed counting logic and the 2D-BMC strategy and demonstrated its usefulness concerning benchmarks from MCC. To the best of our knowledge, DCModelChecker is the only tool that uses 2D-BMC for Petri Nets, specifying properties in an extension of linear temporal logic while utilizing an SMT solver and additionally providing a counterexample trace. Future research directions include optimizing our tool by implementing an efficient linear size encoding in the bound $k$ for the logic $\mathcal{L}$. The BMC technique only provides the counterexamples when the property is unsatisfiable. It would be worth the effort to leverage inductive reasoning to have a full decision procedure, for when the property is satisfiable. QCOVER \cite{7} and ICOVER \cite{14} already have implemented such a procedure for a specific set of properties of Petri Nets. As part of future work, we would like to establish the completeness thresholds of BMC \cite{11}. Another work in progress is an extension of $\mathcal{L}_C$ to account for identifiable clients, which will enable us to verify richer infinite-state systems.

References

A Case Study: Autonomous Parking System

In crowded urban spaces, parking one’s vehicle can be a hassle, given that there is no guarantee of finding an available parking lot in a reasonable amount of time. There have been various approaches to resolve this, from the industry as well as academia [29,9]. The automobile industry has implemented quite successfully, the autonomous parking system, wherein the idea is that the vehicle can be guided into a parking lot, without the need for a valet through communication between the vehicle and the parking space environment. This is especially useful in smart city projects, and futuristically, the driverless vehicle eco system [16].

In this section, we explore how to verify an Autonomous Parking System (APS) where the number of parking requests is unbounded. This is an example from client-server paradigm, more specifically, a single server multiple client system—the parking system is a server whereas the vehicles (say, cars) requesting parking lots are clients. This can be represented by a state-transition system as given Fig. 4 and Fig. 5 and consequently, as a Petri Net as in Fig. 6.

In any configuration (marking) of the Petri Net execution, the number of tokens in any place corresponds to those many clients (cars) in that state. We will now look at the formal description of the APS as a net and express its properties using $L_C$.

A.1 Introduction to Autonomous Parking System

Typically, any parking space in a busy area, such as the city square, or the airport, services hundreds of vehicles. It may employ valets to assist the drivers. It is often the case, that the parking space may not be available, and navigating to a parking lot can be cumbersome. Finding a parking lot and arriving at it, is a process that can be autonomous. The system itself can be made autonomous, by using technology to track the vehicles and respond to requests for parking. In this case study, the autonomous parking system is the server, and the vehicles making requests for parking are the clients. Theoretically, there could be an unbounded number of clients. We assume that there is enough parking space available, to demonstrate this unbounded client-server system.

Modeling of Autonomous Parking System using Petri Nets. We will now model the interactions between the parking system and the vehicle requesting parking space as in Fig. 4 and Fig. 5. Initially, the system is ready to service the requests, which is the $server_{ready}$ state. We assume a steady inflow of requests for parking, to keep the system sufficiently occupied. When a vehicle inquires for parking space, the vehicle is in the $parking_{requested}$ state. Now, the system may non-deterministically choose to either grant or reject the parking request based on local information such as space availability, the priority of incoming requests, and the like. We assume two disjoint workflows for each scenario. Firstly, if the system accepts the request, the system is in $request_{granted}$ state and simultaneously, the vehicle goes to $occupy_{parking_{lot}}$ state. At some point, the vehicle...
gives up its allocated parking space, is in \textit{exit\_parking\_lot\_successfully} and simultaneously the system is in \textit{deallocate\_parking\_lot} state. This marks the successful exit of the vehicle from the system. Secondly, if the system rejects the request, the vehicle is in \textit{parking\_unavailable} state and the system is in \textit{request\_rejected} state. The only option is for the vehicle to exit. At any point, the system can either accept or reject the request, after granting the request, the server can go to \textit{server\_busy} state. Theoretically, this description allows for an unbounded number of vehicle requests to be processed by the system, albeit there may be limitations on the availability of the parking space. We assume that the autonomous parking system can reasonably guide the vehicle manoeuvres within the parking lot. It is not difficult to observe that the combined interactions between the system and vehicles described above can be interleaved and modeled as a single Petri Net as in Fig. 6. In this Petri Net, the successful scenario is marked with a green rectangle, and the places $p_0$, $p_2$ and $p_4$ correspond to the vehicle states \textit{parking\_requested}, \textit{occupy\_parking\_lot} and \textit{exit\_parking\_lot}, the places $p_1$, $p_7$, $p_3$ and $p_5$ correspond to the server states \textit{server\_ready}, \textit{server\_busy}, \textit{request\_granted} and \textit{deallocate\_parking\_lot}, transitions $t_1$, $t_2$, $t_3$ represent the request, entry and exit of the vehicle respectively. The firing of transition $t_0$ acts as the source, where an unbounded number of vehicle requests can be spawned. The firing of transition $t_6$ represents the server completing the processing of a request and returning to its ready state. In the unsuccessful scenario, marked with a red rectangle, the transition $t_4$ is fired when the server rejects the request, which brings the vehicle to \textit{parking\_unavailable} represented by place $p_6$, and the server goes to \textit{request\_rejected} represented by place $p_8$. The firing of transition $t_7$ ensures that the rejected request is processed by the server, and it is ready to take on more vehicle requests. Notice that the initial marking of the system is given as follows, assuming an ordering on the places $M_0 = (0,1,0,0,0,0,0,0)$, where there is exactly one token in place $p_1$ indicating the availability of the server. In any marking of the net, there is exactly one token in either of the server places $p_1$, $p_7$, or $p_8$ due to the structure of the net.

![State diagram of autonomous parking system (server)](image-url)
The standard representation of the Petri Net is the PNML format [15]. We made use of Wolfgang Tool [30], a Petri Net editor to draw and visualize the configurations of the net. The PNML file for APS and the executable (jar file) for the tool are available in the artifact.

A.2 Expressing properties in $\mathcal{L}_C$ and the results of 2D-BMC

1. Consider the property of the APS where there are equal number of accept responses by the server to the number of accepted parking requests by the clients. This can be paraphrased as always the number of tokens in place $p_2$
equals the number of tokens in \( p_3 \). Consider the following property written
in \( 
L_C \):
\[
G((\#x)p_2(x) > p_3(x))(\#x)p_3(x) > p_2(x))
\]
Our algorithm looks for a counterexample trace. This is satisfiable initially,
at \( k = 0 \) where \( \kappa = 1 \) (number of tokens) and \( \lambda = 0 \) (time instance).

2. \( F((\#x)p_2(x) <= p_3(x) \& (\#x)p_3(x) <= p_2(x)) \)

We expect that this formula will be true for all bounds. Hence while having
bound \( k = 5 \) or \( k = 100 \), we get the same output on reaching the bound,
without any counterexample.

3. Consider the property, where there is atleast one token in either place \( p_1 \)
or \( p_7 \) or \( p_8 \). From the net, \( p_1 + p_7 + p_8 = 1 \) is an invariant. These places
represent the server being available, busy and server rejecting the request.
\[
G((\#x > 0)p_1(x)|(\#x > 0)p_7(x)|(\#x > 0)p_8(x)).
\]
On verifying with our tool, we do not find any counterexample for this prop-
erty for bound \( k = 5 \).

4. Comparison between the number of rejected requests and accepted requests.
\( F((\#x)p_6(x) > p_2(x)) \).
At \( k = 4 \), \( \kappa = 1 \), \( \lambda = 3 \) we encounter a loop, and the property is unsatisfiable
for all paths of length \( k = 4 \).

5. Consider this property using the Until operator, essentially this is \( true \ U p_2 = p_1 \).
\[
((\#x > 0)p_1(x)|(\#x > 0)p_7(x)|(\#x > 0)p_8(x))U((\#x)p_2(x) <= p_3(x) \& (\#x)p_3(x) <= p_2(x)).
\]
As expected, this property holds true for bound \( k = 5 \), and no coun-
terexample is found.
B A Case Study: Unbounded Process Scheduler

In this section, we explore how to verify a process scheduler with unbounded processes in an operating system. This is an example from the client-server paradigm: the scheduler is a server whereas the processes requesting CPU time are clients. Each process has a behaviour that can be represented by a state-transition system [18] as given in the Fig. 7.

Unbounded Process Scheduler (UPS), that we have considered in our paper, is a kind of single server multiple client system, where one server and an unbounded number of clients collaborate. In the process model we map each state from \{new, runnable, running, blocked, terminated\} to a place in the Petri Net. In any configuration (marking) of the Petri Net execution, the number of tokens in any place corresponds to as many UPS clients in that state. We will now look at the formal description of the UPS as a net and express its properties using \(L_C\).

![Fig. 7. State diagram of the basic model for processes within an operating system](image)

B.1 Introduction to Unbounded Process Scheduler

Processes are programs in execution. Typically, a process changes its state dynamically based on internal and external inputs. When a process is created, it is in the new state. When it is waiting to be assigned a processor, it is in ready state, alternatively called as runnable state. When instructions are being executed in the process, it is running. The running process may be interrupted by external events such as waiting for keyboard input when

![Fig. 8. Petri net for UPS](image)
it gets \textit{blocked}. On event completion, \textit{blocked} process goes to \textit{runnable}, and waits for its turn to run. A \textit{running} process may complete its execution and \textit{terminate}. This is usually known as the lifecycle of a process in a process scheduler given in Fig. [7]

\textbf{Modeling of Unbounded Process Scheduler using Petri Nets.} It is not difficult to observe that the unbounded process scheduler described above can be modeled as a Petri Net as in Fig. [8]. The initial marking here is given as follows, assuming an ordering on the places $M_0 = (0, 0, 0, 0, 0)$. Here each token corresponds to a process. Transition $t_0$ is used to spawn new processes, while transition $t_7$ is used to take out the processes, terminated by transition $t_6$. Transition $t_1$ is used to load the new processes, transition $t_2$ is to used to run, while $t_3$ is used to deschedule it. Transition $t_4$ corresponds to blocking a process, while $t_5$ corresponds to interrupt. The standard representation of the Petri Net is the PNML format [15]. A Petri Net Editor such as Wolfgang Tool [30] can be used to view existing PNML files as a graphical representation as well as execute the enabled transitions and view the run of the Petri Net. Additionally, Wolfgang allows us to draw a Petri Net of our own. We drew our own Petri Net, using Wolfgang [30] for the UPS case study.

\textbf{B.2 Expressing properties in $L_C$ and the results of 2D-BMC}

The following are the descriptions of the properties of the UPS described above, along with the results from Table. [2]

1. Consider the property of the UPS, $F(\#x)p_1(x) > p_0(x)$ which expresses that at some time, the number of runnable processes at place $p_1$ is greater than the number of newly created processes at place $p_0$. This is satisfiable at time instance $\lambda = 3$, when there is exactly one runnable process in place $p_1$ leading to $\kappa = 1$ and place $p_0$ does not contain any processes.

2. Consider the property $G(\#x)p_2(x) \leq p_1(x) \leq p_0(x)$ which states that it is always the case that the number of running processes is less than or equal to the number of runnable processes and the number of runnable processes is less than or equal to created processes. This is satisfiable at time instance $\lambda = 2$ and $\kappa = 1$.

3. Consider the property $G(t_0 \& t_1 \& t_7)$ which states that it is always the case that process creation, process loading into memory, and process scheduled to run occur simultaneously. This is satisfiable at $\kappa = 0$, $\lambda = 0$.

4. Consider the property $F(t_0 \& t_1 \& t_7)$ which states that at some time process creation, process loading into memory, and process scheduled to run occur simultaneously. This is unsatisfiable up to bound $k = 5$ and the external termination bound is reached. This indicates that such a scenario is not possible up to the given bound.
5. Consider the property $F(\#x \leq 1)p_2(x) \land (\#x \leq 3)p_1(x) \land (\#x \leq 2)p_0(x)$ which states that at some time there is at most 1 running process and there are at most 3 runnable processes and there are at most 2 newly created processes in the scheduler. This is not satisfiable up to the bound $k = 5$, hence the program exits, saying that the external termination bound is reached.